NAME Solution

STUDENT ID ______

RECITATION INSTRUCTOR ______

RECITATION TIME ______

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DIRECTIONS

1. Write your name, student ID number, recitation instructor's name and recitation time in the space provided above. Also write your name at the top of pages 2–6.
2. The exam has six (6) pages, including this one.
3. Circle the correct answer for problems 1–3. Write your answer in the box provided for problems 4–12.
4. You must show sufficient work to justify your answers.
5. Credit for each problem is given in parentheses in the left hand margin.
6. No books, notes or calculators may be used on this exam.

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(5) 1. Let $\vec{a} = \vec{i} - 2\vec{j} + 3\vec{k}$ and $\vec{b} = 3\vec{i} + 4\vec{j} + 7\vec{k}$. Then $\dfrac{\vec{a}\cdot\vec{b}}{\|\vec{a}\|} =$

$\vec{a}\cdot\vec{b} = 3 - 8 + 21 = 16$

$\|\vec{a}\| = \sqrt{1+4+9} = \sqrt{14}$

A. 8

B. $\dfrac{33}{14}$

C. $\dfrac{33}{\sqrt{14}}$

D. $\dfrac{16}{\sqrt{14}}$

E. $\dfrac{8}{7}$

(7) 2. Symmetric equations for the tangent line to the curve $\vec{r}(t) = e^t\vec{i} + (2t+3)\vec{j} - \sin t\vec{k}$ at the point $(1,3,0)$ are:

$\vec{r}\,'(t) = e^t\vec{i} + 2\vec{j} - \cos t\,\vec{k}$

$\vec{r}\,'(0) = \vec{i} + 2\vec{j} - \vec{k}$

$=$ direction.

A. $\dfrac{x-1}{1} = \dfrac{y-3}{2} = \dfrac{z}{-1}$

B. $\dfrac{x-1}{1} = \dfrac{y-3}{3} = \dfrac{z}{5}$

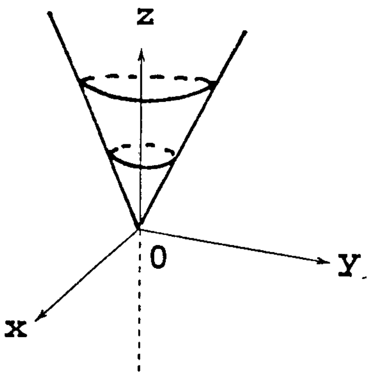
C. $\dfrac{x-1}{e^t} = \dfrac{y-3}{2} = \dfrac{z}{-\cos t}$

D. $x = 1+t,\ y = 3+2t,\ z = -t$

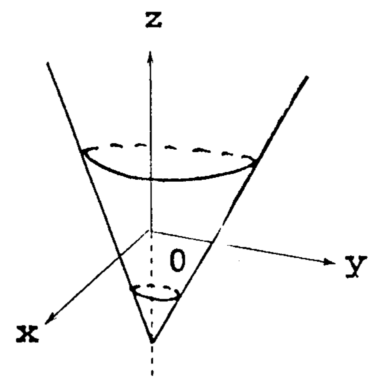
E. $x = 1+t,\ y = 3+3t,\ z = 5t$

(7) 3. Which of the following surfaces represents the graph of $f(x, y) = 4x^2 + y^2 - 4$?

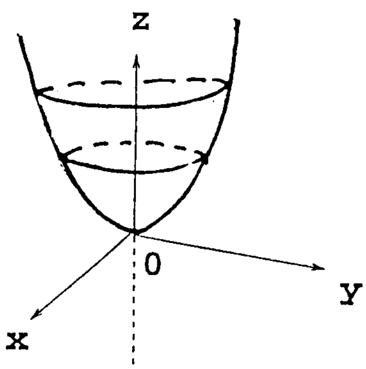
A



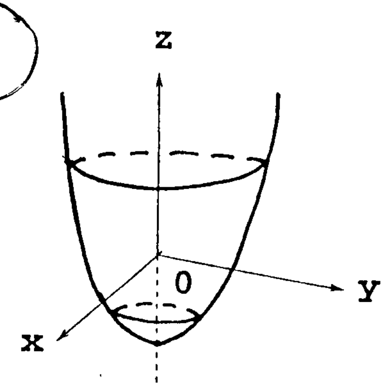
B



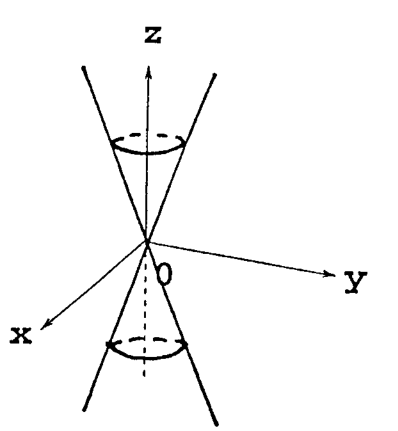
C



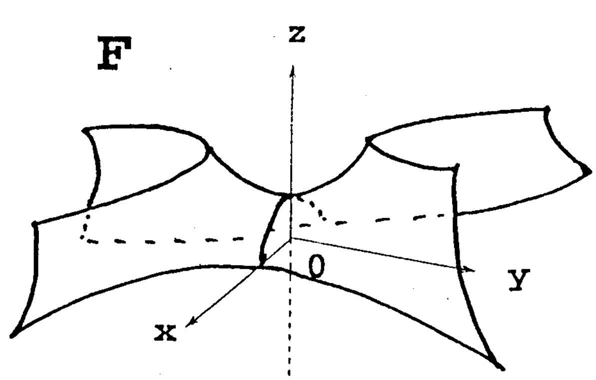
D



E



F



(9) 4. Find an equation of the plane through the points $\overset{P}{(1,2,-3)}$, $\overset{Q}{(4,1,1)}$, and $\overset{R}{(5,0,2)}$.

$$\overrightarrow{PQ} = 3\vec{i} - \vec{j} + 4\vec{k}, \quad \overrightarrow{PR} = 4\vec{i} - 2\vec{j} + 5\vec{k}$$

$$\overrightarrow{PQ} \times \overrightarrow{PR} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 3 & -1 & 4 \\ 4 & -2 & 5 \end{vmatrix} = 3\vec{i} + \vec{j} - 2\vec{k}$$

Equation of plane:

$$3(x-1) + (y-2) - 2(z+3) = 0$$

or

$$\boxed{3x + y - 2z - 11 = 0}$$

(9) 5. If a particle has velocity $\vec{v}(t) = 2\vec{i} + 3t^2\vec{j} + e^t\vec{k}$ and initial position $\vec{r}(0) = \vec{i} + 2\vec{k}$, find the position $\vec{r}(t)$ of the particle at time t.

$$\vec{r}(t) = 2t\vec{i} + t^3\vec{j} + e^t\vec{k} + \vec{C}$$

$$\vec{r}(0) = \vec{k} + \vec{C} = \vec{i} + 2\vec{k}$$

$$\therefore \vec{C} = \vec{i} + \vec{k}$$

$$\vec{r}(t) = 2t\vec{i} + t^3\vec{j} + e^t\vec{k} + \vec{i} + \vec{k}$$

or

$\vec{r}(t) = \boxed{(2t+1)\vec{i} + t^3\vec{j} + (e^t+1)\vec{k}}$

(9) 6. If $w = f(t^2, 2t^3)$, where $f(x,y)$ is differentiable, $f_x(1,2) = 5$ and $f_y(1,2) = 8$, compute $\frac{dw}{dt}$ at $t = 1$.

$$\frac{dw}{dt} = f_x \frac{dx}{dt} + f_y \frac{dy}{dt}$$

$$w = f(x,y), \quad x = t^2, \quad y = 2t^3$$

$$\frac{dw}{dt} = f_x \cdot 2t + f_y \, 6t^2$$

$$\left.\frac{dw}{dt}\right|_{t=1} = 5 \cdot 2 + 8 \cdot 6 = 58$$

$\left.\frac{dw}{dt}\right|_{t=1} = \boxed{58}$

(9) 7. Find the directional derivative of $f(x,y) = \frac{1}{3}x^3 + x \ln y$ at the point $(2,1)$ in the direction from $(2,1)$ to $(5,5)$.

$$D_{\vec{u}} f = \operatorname{grad} f \cdot \vec{u}$$

$$\operatorname{grad} f = f_x \vec{i} + f_y \vec{j}$$

$$= (x^2 + \ln y)\vec{i} + \frac{x}{y}\vec{j}$$

$$\operatorname{grad} f(2,1) = 4\vec{i} + 2\vec{j}$$

$$\text{Direction} = \vec{a} = 3\vec{i} + 4\vec{j}$$

$$\vec{u} = \frac{\vec{a}}{\|\vec{a}\|} = \frac{3}{5}\vec{i} + \frac{4}{5}\vec{j}$$

$$D_{\vec{u}} f(2,1) = \frac{12}{5} + \frac{8}{5} = 4$$

$D_{\vec{u}} f(2,1) = \boxed{4}$

(9) 8. Find the length, L, of the curve $\vec{r}(t) = 2t\vec{i} + t^2\vec{j} + \ln t\vec{k}$ for $1 \le t \le 2$.

$L = \int_1^2 \|\vec{r}'(t)\| \, dt$, $\vec{r}'(t) = 2\vec{i} + 2t\vec{j} + \frac{1}{t}\vec{k}$

$L = \int_1^2 \sqrt{4 + 4t^2 + \frac{1}{t^2}} \, dt = \int_1^2 \sqrt{(2t + \frac{1}{t})^2} \, dt$

$= \int_1^2 (2t + \frac{1}{t}) dt = t^2 + \ln t \Big]_1^2$

$= 4 + \ln 2 - 1$

$L =$ $\boxed{3 + \ln 2}$

(9) 9. Find an equation of the plane tangent to the graph of $f(x,y) = \dfrac{x+1}{y-1}$ at the point $(3,2,4)$.

$f_x = \frac{1}{y-1}$, $f_y = \frac{-(x+1)}{(y-1)^2}$

$f_x(3,2) = 1$, $f_y(3,2) = -4$

Tan plane : $f_x(3,2)(x-3) + f_y(3,2)(y-2) - (z-4) = 0$

$(x-3) - 4(y-2) - (z-4) = 0$

or

tangent plane: $\boxed{x - 4y - z + 9 = 0}$

(9) 10. Find the critical point(s) of $f(x,y) = (\sin x)(\cos y)$ in the square, $0 \le x \le \pi$, $0 \le y \le \pi$.

$f_x = \cos x \cos y$, $f_y = -\sin x \sin y$

$f_x = 0 \rightarrow x = \frac{\pi}{2}$ or $y = \frac{\pi}{2}$

$f_y = 0 \rightarrow x = 0$ or π, or $y = 0$ or π

$\therefore$ If $x = \frac{\pi}{2}$, $y = 0$ or π

if $x = 0$ or π, $y = \frac{\pi}{2}$

(9) 11. Apply the second partial derivative test to determine whether

$$f(x,y) = x^3 + y^3 - xy - 2x - 2y$$

has a relative maximum, a relative minimum, or a saddle point at its critical point (1, 1). Circle the correct answer. (Give reasons for your answer.)

Relative Maximum

Relative Minimum

Saddle Point

$f_x = 3x^2 - y - 2$; $f_y = 3y^2 - x - 2$.

$f_{xx} = 6x$, $f_{yy} = 6y$, $f_{xy} = -1$

$f_{xx}(1,1) = 6$ $f_{yy}(1,1) = 6$, $f_{xy}(1,1) = -1$

$D(1,1) = f_{xx}(1,1) \cdot f_{yy}(1,1) - [f_{xy}(1,1)]^2$

$= 6 \cdot 6 - 1 = 35 > 0$.

and $f_{xx}(1,1) > 0$

$\therefore$ (1,1) is a relative minimum

(9) 12. Find the maximum value of $f(x, y) = x^2 - 6y$ on the circle $x^2 + y^2 = 25$. (Give reasons for your answer.)

Method 1 $g(x,y) = x^2 + y^2 = 25$

grad f $= \lambda$ grad g.

$2x = \lambda 2x$

$-6 = \lambda 2y$.

Solutions

$x = 0$, $(x^2+y^2=25)$ $y = \pm 5$

if $x \neq 0$, $\lambda = 1$, since $-6 = \lambda 2y$,

$y = -3$ $(x^2+y^2=25)$ $x = \pm 4$.

points $(0,5)$ $(0,-5)$ $(\pm 4, -3)$.

$f(0,5) = -30$

$f(0,-5) = 30$

$\boxed{f(\pm 4, -3) = 34}$ Max

Method 2

$x^2 = 25 - y^2$

$\therefore f(x,y) = 25 - y^2 - 6y$

$\frac{df}{dy} = -2y - 6$

$\frac{df}{dy} = 0 \rightarrow y = -3$

$x^2 + y^2 = 25 \longrightarrow x = \pm 4$

Test $\frac{d^2f}{dy^2} = -2 < 0$ $\therefore$

rel Max

$f(\pm 4, -3) = 34$

Maximum Value: 34