MA 26100
EXAM 1 Form 01
February 21, 2017

NAME ______________________ YOUR TA'S NAME ______________________

STUDENT ID # ____________________ RECITATION TIME ____________________

1. You must use a #2 pencil on the mark–sense sheet (answer sheet).

2. On the scantron, write 01 in the TEST/QUIZ NUMBER boxes and blacken in the appropriate spaces below.

3. On the scantron, fill in your TA's name and the course number.

4. Fill in your NAME and STUDENT IDENTIFICATION NUMBER and blacken in the appropriate spaces. BE SURE TO INCLUDE THE TWO LEADING ZEROS.

5. Fill in your four-digit SECTION NUMBER. If you do not know your section number, please ask your TA.

6. Sign the scantron.

7. Fill in your name and your instructor's name on the question sheets above.

8. There are 12 questions, each worth 8 points (you will automatically earn 4 points for taking the exam). Blacken in your choice of the correct answer in the spaces provided for questions 1–12. Do all your work on the question sheets.

9. Turn in both the scantron and the exam booklet when you are finished.

10. You cannot turn in your exam during the first 20 min or the last 10 min of the exam period.

11. NO CALCULATORS, PHONES, BOOKS, OR PAPERS ARE ALLOWED. Use the back of the test pages for scrap paper.

EXAM POLICIES

1. Students may not open the exam until instructed to do so.
2. Students must obey the orders and requests by all proctors, TAs, and lecturers.
3. No student may leave in the first 20 min or in the last 10 min of the exam.
4. Books, notes, calculators, or any electronic devices are not allowed on the exam, and they should be put away and should not be visible at all. Students may not look at anybody else's test, and may not communicate with anybody else except, if they have a question, with their TA or lecturer.
5. After time is called, the students have to put down all writing instruments and remain in their seats, while the TAs will collect the scantrons and the exams.
6. Any violation of these rules and any act of academic dishonesty may result in severe penalties. Additionally, all violators will be reported to the Office of the Dean of Students.

I have read and understand the exam rules stated above:

STUDENT NAME: ____

STUDENT SIGNATURE: ____

1. Let L be the line that is parallel to the planes

$$x - y + z = 1 \text{ and } 2x + y + z = 4$$

and that passes through $(1, 3, -3)$. At what point does L intersect the plane $z = 0$?

A. $(1, -2, 0)$
B. $(2, 1, 0)$
C. $(1, 3, 0)$
D. $(-2, 2, 0)$
E. $(-1, 4, 0)$

The given planes have normal vectors $n_1 = \langle 1, -1, 1 \rangle$ and $n_2 = \langle 2, 1, 1 \rangle$.

L has direction vector $n_1 \times n_2$

$$= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & -1 & 1 \\ 2 & 1 & 1 \end{vmatrix} = \vec{i}(-2) - \vec{j}(-1) + \vec{k}(3) = \langle -2, 1, 3 \rangle$$

L: $\vec{r}(t) = \langle 1, 3, -3 \rangle + t\langle -2, 1, 3 \rangle = \langle 1-2t, 3+t, -3+3t \rangle$

Solve $-3 + 3t = 0$
$t = 1$

$$\boxed{\vec{r}(1) = \langle -1, 4, 0 \rangle}$$

2. Which of the following surfaces describes the surface $4x^2 - y^2 - z^2 = 4$?

A. elliptic paraboloid
B. hyperboloid of one sheet
C. hyperboloid of two sheets
D. cone
E. hyperbolic paraboloid

$$x^2 - \frac{y^2}{4} - \frac{z^2}{4} = 1$$

Surface is either an ellipse or a hyperboloid.

Two negatives indicate $\boxed{\text{hyperboloid of 2 sheets}}$

3. Suppose the trajectories of two particles are given by

$$\mathbf{r}_1(t) = \langle t+1, 2t^{1/2}, 2^{1/2}t\rangle,$$
$$\mathbf{r}_2(t) = \langle 2t, t^2+1, t^2-2t+2^{1/2}+1\rangle.$$

Find the angle between their tangent vectors at their point of collision.

A. 0
B. $\frac{\pi}{6}$
C. $\frac{\pi}{4}$
D. $\frac{\pi}{3}$
E. $\frac{\pi}{2}$

Collision occurs when $\begin{cases} 2t = t+1 \\ 2t^{1/2} = t^2+1 \\ 2^{1/2}t = t^2-2t+2^{1/2}+1 \end{cases}$ $t=1.$

$\vec{r}_1{}'(t) = \langle 1, t^{-1/2}, 2^{1/2}\rangle$ $\quad \vec{r}_2{}'(t) = \langle 2, 2t, 2t-2\rangle$

$\vec{r}_1{}'(1) = \langle 1, 1, 2^{1/2}\rangle$ $\quad \vec{r}_2{}'(1) = \langle 2, 2, 0\rangle$

$$\cos\theta = \frac{\langle 1,1,2^{1/2}\rangle \cdot \langle 2,2,0\rangle}{|\langle 1,1,2^{1/2}\rangle|\,|\langle 2,2,0\rangle|} = \frac{2+2}{\sqrt{1+1+2}\sqrt{4+4}} = \frac{4}{\sqrt{4}\sqrt{8}} = \frac{4}{4\sqrt{2}} = \frac{1}{\sqrt{2}}$$

$\boxed{\theta = \pi/4}$

4. A particle moves with acceleration $\mathbf{a}(t) = e^t\mathbf{k}$ and initial velocity and position given by $\mathbf{v}(0) = \mathbf{0}$ and $\mathbf{r}(0) = \mathbf{j}+\mathbf{k}$, respectively. Where is the particle at time $t = 2$?

A. $(0, 1, e^2-2)$
B. $(0, 1, e^2)$
C. $(0, 1, e-1)$
D. $(1, 1, e^2-2)$
E. $(1, 1, e^2)$

$\vec{v}(t) = \int \vec{a}(t)\,dt = \int e^t\vec{k}\,dt = e^t\vec{k} + \vec{c}$

$\vec{0} = \vec{v}(0) = \vec{k} + \vec{c}, \quad \vec{c} = -\vec{k}.$

$\vec{v}(t) = e^t\vec{k} - \vec{k} = (e^t-1)\vec{k}$

$\vec{r}(t) = \int \vec{v}(t)\,dt = \int (e^t-1)\vec{k}\,dt$

$= (e^t - t)\vec{k} + \vec{d}$

$\vec{j} + \vec{k} = \vec{r}(0) = (1-0)\vec{k} + \vec{d}.$

$\vec{d} = \vec{j}$

$\vec{r}(t) = (e^t - t)\vec{k} + \vec{j}.$

$\vec{r}(2) = \vec{j} + (e^2-2)\vec{k} = \boxed{\langle 0, 1, e^2-2\rangle}$

5. A particle moves along according to $\mathbf{r}(t) = \langle 5t, 1-3t, 5+4t \rangle$, $t \geq 0$. What is the x-coordinate of the particle after it has traveled a distance of 2 units along the curve?

A. 10

B. $\sqrt{2}$

C. $5\sqrt{2}$

D. $\frac{1}{10}$

E. $\frac{5\sqrt{2}}{2}$

$s(t) = \int_0^t |\vec{r}\,'(u)|\, du = \int_0^t |\langle 5, -3, 4 \rangle|\, du$

$= \int_0^t \sqrt{25+9+16}\, du = \int_0^t \sqrt{50}\, du = \sqrt{50}\, u \Big|_{u=0}^{u=t}$

$= \sqrt{50}\, t$

$t = \frac{s}{\sqrt{50}}$. If $s = 2$, then $t = \frac{2}{\sqrt{50}} = \frac{2}{5\sqrt{2}} = \frac{\sqrt{2}}{5}$

$\vec{r}(\sqrt{2}/5) = \langle 5(\sqrt{2}/5), \ , \ \rangle$

$\boxed{\text{x-coordinate is } \sqrt{2}}$.

6. Consider the limits

$$\mathrm{I} = \lim_{(x,y)\to(0,0)} \frac{3x-2y}{\sqrt{x^2+y^2}} \quad \text{and} \quad \mathrm{II} = \lim_{(x,y)\to(0,0)} \frac{e^{x+y}}{1+e^{x-y}}.$$

Which one of the following statements is true?

A. I = 3 and II = 1

B. both I and II do not exist

C. I does not exist and II = $\frac{1}{2}$

D. I does not exist and II = 1

E. I = 1 and II = 1

I:

Along $y = x$ $\quad \frac{3x-2y}{\sqrt{x^2+y^2}} = \frac{3x-2x}{\sqrt{x^2+x^2}} = \frac{x}{\sqrt{2x^2}}$

$= \frac{x}{\sqrt{2}\,|x|} = \begin{cases} \frac{1}{\sqrt{2}}, & x > 0 \\ \frac{-1}{\sqrt{2}}, & x < 0 \end{cases}$

$\boxed{\text{I does not exist}}$, because limit along $y=x$ does not exist.

II: Sub $(x,y) = (0,0)$

$\frac{e^{0+0}}{1+e^{0-0}} = \boxed{\frac{1}{2}}$.

7. The domain of the function $f(x, y) = \ln(\frac{x}{y+2})$ is

A. $y \neq -2, x > 0$

B. $y > -2, x > 0$ or $y < -2, x < 0$

C. $y > -2, x > 0$

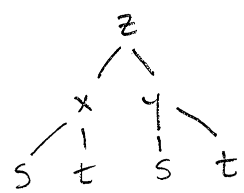
D. $y > 0, x > 0$

E. $y > 0, x > 0$ or $y < -2, x < 0$

$\frac{x}{y+2} > 0$. $\quad (x > 0$ and $y+2 > 0)$ or $(x < 0$ and $y+2 < 0)$

8. Let $z = f(x, y)$ where $f_x = xy^2$ and $f_y = x^2y + y^2$. If $x = s + 2t$ and $y = t^2$ find z_t at $(s, t) = (2, 1)$.

$z_t = f_x \frac{\partial x}{\partial t} + f_y \frac{\partial y}{\partial t} = 4(2) + 17(2) = \boxed{42}$



A. 14

B. 21

C. 8

D. 42

E. 32

When $(s,t) = (2,1)$

$\frac{\partial x}{\partial t} = 2$, $\frac{\partial y}{\partial t} = 2t = 2$, $x = 2+2 = 4$, $y = 1^2 = 1$.

$f_x = (4)1^2 = 4$, $f_y = 4^2(1) + 1^2 = 17$

9. If $f(x, y) = \ln(xy^2 + x)$ find f_{xy}.

A. 0

B. $\frac{-y^2-1}{(xy^2+x)^2}$

C. $\frac{-2y}{(xy^2+x)^2}$

D. $\frac{-y^2}{(xy^2+x)^2}$

E. $\frac{-2xy}{(xy^2+x)^2}$

$$f_x = \frac{1}{xy^2+x} \cdot (y^2+1) = \frac{\cancel{y^2+1}}{x(\cancel{y^2+1})} = \frac{1}{x}$$

$$f_{xy} = 0$$

10. If $z = \frac{9}{u+v^2}$ then the tangent plane to the graph at $u = 2$, $v = 1$ is

A. $-3z + 2u - v = -6$

B. $3z + 2u + v = 6$

C. $z - u + 2v = 3$

D. $z + u + 2v = 7$

E. $z + 9u - 2v = 19$

$$z - z_0 = f_u (u - u_0) + f_v (v - v_0)$$

$$z - 3 = -(u-2) - 2(v-1)$$

$$z - 3 = -u + 2 - 2v + 2$$

$$\boxed{z + u + 2v = 7}$$

$$z = f(u,v) = \frac{9}{u+v^2}$$

$$f_u = -9(u+v^2)^{-2}$$

$$f_u(2,1) = -9(2+1^2)^{-2} = -9 \cdot 3^{-2} = -1$$

$$f_v = -9(u+v^2)^{-2}(2v)$$

$$f_v(2,1) = -9 \cdot 3^{-2}(2) = -2$$

7

$$z_0 = f(u_0, v_0) = f(2,1) = \frac{9}{2+1} = 3$$

11. Suppose the graph of $z = g(x, y)$ intersects the plane $x = 0$ along the curve $z = y^3+2y^2+1$. What is $g_y(0, 2)$?

A. 1

B. 4

C. 8

D. 17

E. 20

$$g_y(x,y) = \frac{\partial z}{\partial y} = 3y^2 + 4y$$

$$g_y(0,2) = 3(2)^2 + 4(2) = 12 + 8 = 20$$

12. Let $x^2 - y^3 + z^2 - z = 12$. If this equation defines z implicitly as a function of x and y find z_y at $(x, y, z) = (3, -1, 2)$.

A. 1

B. $\frac{3}{4}$

C. -3

D. 3

E. $-\frac{3}{4}$

$$\frac{\partial}{\partial y}\left(x^2 - y^3 + z^2 - z\right) = \frac{\partial}{\partial y}(12)$$

$$-3y^2 + 2z\frac{\partial z}{\partial y} - \frac{\partial z}{\partial y} = 0$$

Sub $(x,y,z) = (3,-1,2)$

$$-3(-1)^2 + 2(2)\frac{\partial z}{\partial y} - \frac{\partial z}{\partial y} = 0.$$

$$4\frac{\partial z}{\partial y} - \frac{\partial z}{\partial y} = 3$$

$$3\frac{\partial z}{\partial y} = 3$$

$$\boxed{\frac{\partial z}{\partial y} = 1}$$