MA261 — EXAM II — FALL 2014 — NOVEMBER 13, 2014
TEST NUMBER 01– GREEN– USE GREEN SCANTRON

INSTRUCTIONS:

1. **Do not open the exam booklet until you are instructed to do so.**
2. There are 7 different test pages (including this cover page). Once you are allowed to open the exam, make sure you have a complete test.
3. Do any necessary work for each problem on the space provided or on the back of the pages of this test booklet. Circle your answers in this test booklet.
4. The number of points each problem is worth is stated next to it. The maximum total is 96 points. Four points will be added for maximum score of 100 points. No partial credit.
5. Make sure the color of your scantron matches the color of the cover page of your exam.
6. Use a # 2 pencil to fill in the required information in your scantron and fill in the circles.
7. Use a # 2 pencil to fill in the answers on your scantron.
8. After you have finished the exam, hand in your scantron and your test booklet to your recitation instructor.

RULES REGARDING ACADEMIC DISHONESTY:

1. Do not leave the exam room during the first 20 minutes of the exam.
2. If you do not finish your exam in the first 50 minutes, you must wait until the end of the exam period to leave the room.
3. Do not seek or obtain any kind of help from anyone to answer questions on this exam. If you have questions, consult only your instructor.
4. Do not look at the exam of another student. You may not compare answers with anyone else or consult another student until after you have finished your exam, handed it in to your instructor and left the room.
5. Do not consult notes, books, calculators.
6. Do not handle phones or cameras, or any electronic device until after you have finished your exam, handed it in to your instructor and left the room.
7. After time is called, the students have to put down all writing instruments and remain in their seats, while the TAs will collect the scantrons and the exams.
8. Anyone who violates these instructions will have committed an act of academic dishonesty. Penalties for academic dishonesty can be very severe and may include an F in the course. All cases of academic dishonesty will be reported immediately to the Office of the Dean of Students.

I have read and understand the above statements regarding academic dishonesty:

STUDENT NAME: ANSWERS

STUDENT SIGNATURE: ____________________

STUDENT ID NUMBER: ____________________

SECTION NUMBER ____________________

RECITATION INSTRUCTOR: ____________________

1. (8 points) The x-coordinate of the point on the plane $z = x + y$ that is closest to the point $(1, 1, 1)$ is equal to

A. $\frac{1}{3}$

B. $\boxed{\frac{2}{3}}$

C. $\frac{3}{4}$

D. $\frac{5}{4}$

E. $\frac{1}{4}$

Square of Distance $g(x,y) = (x-1)^2 + (y-1)^2 + (z-1)^2$

Constraint: $f = z - x - y = 0$.

Lagrange multipliers: $\nabla g = \lambda \nabla f$.

$$(2(x-1), 2(y-1), 2(z-1)) = \lambda(-1,-1,1)$$

$$x-1 = -\frac{\lambda}{2};\quad y-1 = -\lambda/2;\quad z-1 = \lambda/2.$$

$$z = x+y. \qquad 1+\lambda/2 = (1-\lambda/2)+(1-\lambda/2)$$

$$1+\lambda/2 = 2-\lambda;\quad 3\lambda/2 = 1, \quad \boxed{\lambda = 2/3}$$

$$x = 1-\lambda/2 = 1-1/3 = 2/3.$$

2. (8 points) Compute $\int\int_D \sqrt{1-y^2}\,dA$, where D is the region of the plane bounded by the lines $x = 0$, $y = x$ and $y = 1$.

A. $\boxed{\frac{1}{3}}$

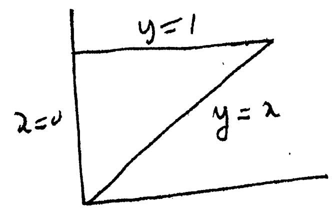
B. $\frac{3}{2}$

C. $\frac{3}{4}$

D. $\frac{1}{4}$

E. $\frac{7}{4}$

Region



$$\iint_D \sqrt{1-y^2}\,dA = \int_0^1\int_0^y \sqrt{1-y^2}\,dx\,dy$$

$$= \int_0^1 (1-y^2)^{1/2}\, y\,dy = \frac{1}{2}\int_0^1 u^{1/2}\,\frac{du}{2}$$

$1-y^2 = u$; $du = -2y\,dy$

$$= \frac{1}{2}\cdot\frac{2}{3}\,u^{3/2}\Big|_0^1 = 1/3.$$

3. **(8 points)** Find the area of the region in the first quadrant inside the circle $x^2+y^2=x$ and outside the circle $x^2+y^2=\frac{1}{2}$.

$x^2+y^2=x$, $(x-\frac{1}{2})^2+y^2=\frac{1}{4}$; $r^2=r\cos\theta$
$r=\cos\theta$

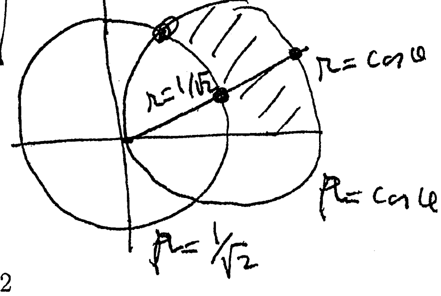
$\boxed{\text{A. } \frac{1}{8}}$

B. $\frac{1}{4}$

C. $\frac{\pi}{4}$

D. $\frac{\sqrt{3}}{16}$

E. $\frac{\pi+2}{4}$



$$A=\int_0^{\pi/4}\int_{1/\sqrt{2}}^{\cos\theta} r\,dr\,d\theta=\frac{1}{2}\int_0^{\pi/4}(\cos^2\theta-\frac{1}{2})\,d\theta$$

$$=\frac{1}{4}\int_0^{\pi/4}\cos 2\theta\,d\theta=$$

$$=\frac{1}{8}\sin 2\theta\Big|_0^{\pi/4}=\frac{1}{8}$$

$\frac{1}{\sqrt{2}}=\cos\theta,\ \theta=\pi/4.$

$\cos 2\theta=\cos^2\theta-\sin^2\theta=2\cos^2\theta-1$

$\cos^2\theta-\frac{1}{2}=\frac{1}{2}\cos 2\theta$

4. **(8 points)** The surface area of the part of the plane $2x+y+z=9$ in the first quadrant above the plane $z=5$ is equal to

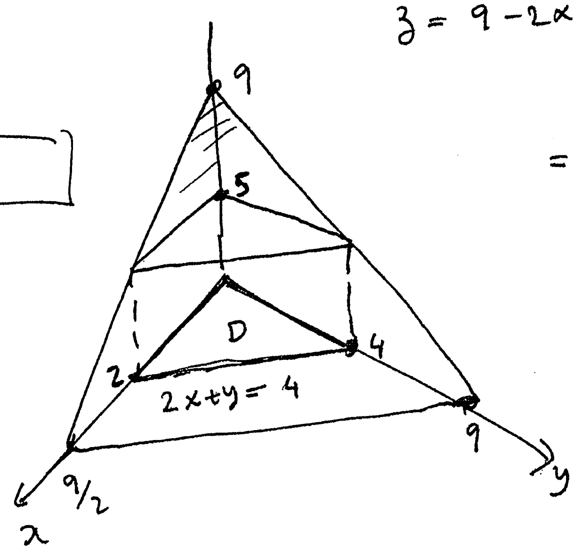
A. $\frac{81}{4}\sqrt{6}$

B. $\frac{25}{4}\sqrt{6}$

$\boxed{\text{C. } 4\sqrt{6}}$

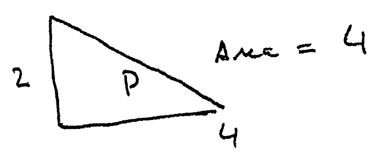
D. $3\sqrt{6}$

E. $2\sqrt{6}$



$z=9-2x-y$ $\quad S=\iint_D\sqrt{1+4+1}\,dA$

$$=\sqrt{6}\iint_D dA=4\sqrt{6}$$



5. **(8 points)** Let D be the region in the first quadrant that is bounded by $x = y^2$, $x = 1$ and $y = 0$. If the density at (x, y) is $\rho(x, y) = 16xy$, and the total mass of D is $m = \frac{8}{3}$, find the x-component of the center of mass of D.

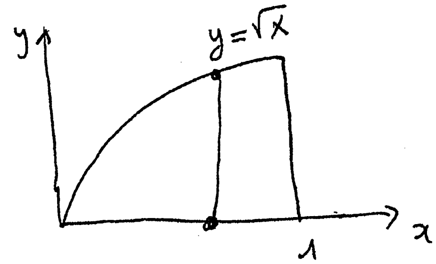
A. $\bar{x} = \frac{1}{2}$

B. $\bar{x} = \frac{1}{4}$

C. $\bar{x} = \frac{3}{4}$

D. $\bar{x} = \frac{2}{3}$

E. $\bar{x} = \frac{5}{6}$



$$\int_0^1 \int_0^{\sqrt{x}} 16x^2y \, dy \, dx = \int_0^1 8x^3 \, dx = 2$$

$$\bar{x} = \frac{2}{8/3} = 2 \times \frac{3}{8} = 3/4.$$

6. **(8 points)** Let

$$I = \int_0^2 \int_0^{\sqrt{4-x^2}} \int_{x^2+z^2}^{4} (3x + z) \, dydzdx.$$

$y = x^2 + z^2$

If we change the order of integration, and integrate first in z then in x and then in y, we find that I is equal to

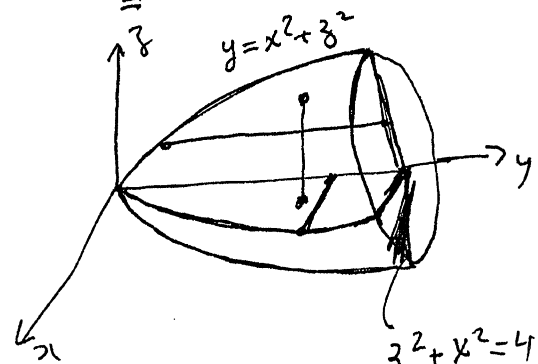
A. $I = \int_0^4 \int_0^{y^2} \int_0^{\sqrt{y-x^2}} (3x + z) \, dzdxdy$

B. $I = \int_0^2 \int_0^{\sqrt{y}} \int_0^{\sqrt{y-x^2}} (3x + z) \, dzdxdy$

C. $I = \int_0^4 \int_0^{y^2} \int_0^{\sqrt{4-y^2-x^2}} (3x + z) \, dzdxdy$

D. $I = \int_0^4 \int_0^{\sqrt{y}} \int_0^{\sqrt{y-x^2}} (3x + z) \, dzdxdy$

E. $I = \int_0^4 \int_0^{y} \int_0^{\sqrt{4-x^2}} (3x + z) \, dzdxdy$



$z^2+x^2=4$

$z^2=4-x^2$

$z=0$ to $z=\sqrt{y-x^2}$

$y=x^2$ $x=0$ to $x=\sqrt{y}$

7. **(8 points)** Let Ω be the region inside the sphere $x^2+y^2+z^2=2$ that lies above the paraboloid $z=x^2+y^2$. Then the volume of Ω is represented by

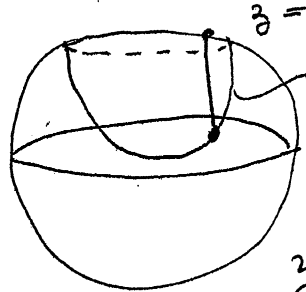
A. $\displaystyle\int_0^{\pi}\int_0^{1}\int_{\sqrt{2-r^2}}^{r^2} r\,dz\,dr\,d\theta$

B. $\displaystyle\int_0^{\pi}\int_0^{2}\int_{r^2}^{\sqrt{2-r^2}} r\,dz\,dr\,d\theta$

C. $\displaystyle\int_0^{2\pi}\int_0^{2}\int_{r^2}^{\sqrt{2-r^2}} r\,dz\,dr\,d\theta$

D. $\displaystyle\int_0^{2\pi}\int_0^{2}\int_{\sqrt{2-r^2}}^{r^2} r\,dz\,dr\,d\theta$

E. $\displaystyle\boxed{\int_0^{2\pi}\int_0^{1}\int_{r^2}^{\sqrt{2-r^2}} r\,dz\,dr\,d\theta}$



$z=\sqrt{2-r^2}$

$z=r^2$

Intersection

$2-r^2=r^4$

$r^4+r^2-2=0$

$(r^2+2)(r^2-1)=0$

$\boxed{r=1}$

$$V=\int_0^{2\pi}\int_0^{1}\int_{r^2}^{\sqrt{2-r^2}} r\,dz\,dr.$$

8. **(8 points)** The integral

$$\int_0^2\int_{-\sqrt{4-y^2}}^{\sqrt{4-y^2}}\int_0^{\sqrt{4-x^2-y^2}}(x^2z+y^2z+z^3)\,dz\,dx\,dy$$ written in spherical coordinates is

A. $\displaystyle\int_0^{\pi}\int_0^{\pi/2}\int_0^{4}\rho^3\cos\phi\sin\phi\,d\rho\,d\phi\,d\theta$

B. $\displaystyle\boxed{\int_0^{\pi}\int_0^{\pi/2}\int_0^{2}\rho^5\cos\phi\sin\phi\,d\rho\,d\phi\,d\theta}$

C. $\displaystyle\int_0^{\pi/2}\int_0^{\pi}\int_0^{4}\rho^3\cos\phi\sin\phi\,d\rho\,d\phi\,d\theta$

D. $\displaystyle\int_0^{\pi/2}\int_0^{\pi/2}\int_0^{2}\rho^5\cos\phi\sin\phi\,d\rho\,d\phi\,d\theta$

E. $\displaystyle\int_0^{\pi}\int_0^{\pi/2}\int_0^{2}\rho^3\cos\phi\sin\phi\,d\rho\,d\phi\,d\theta$

Integrand: $z(x^2+y^2+z^2)$

$=\rho\cos\phi\cdot\rho^2=\rho^3\cos\phi$

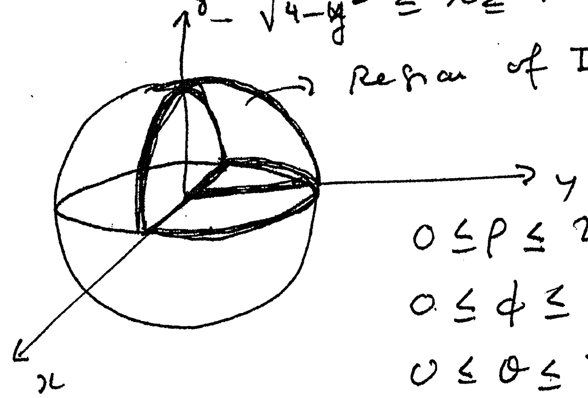
Element of volume: $\rho^2\sin\phi\,d\rho\,d\phi\,d\theta$

Limits of Integration:

$z^2\le 4-x^2-y^2$

$-\sqrt{4-y^2}\le x\le\sqrt{4-y^2}$

Region of Integration



$0\le\rho\le 2$

$0\le\phi\le\pi/2$

$0\le\theta\le\pi.$

9. **(8 points)** The equation for the curve of intersection for the plane $z = 5$ and the sphere

$\rho = 8\cos\phi$ is

A. $x^2 + y^2 = 16$

B. $x^2 + y^2 = 4$

C. $x^2 + y^2 = 9$

D. $\boxed{x^2 + y^2 = 15}$

E. $x^2 + y^2 = 1$

$$\rho^2 = 8\rho\cos\phi = 8z$$

$$x^2+y^2+z^2 = 8z. \qquad z = 5$$

$$x^2+y^2+25 = 40$$

$$\boxed{x^2+y^2 = 15}$$

10. **(8 points)** Find the volume of the solid inside the sphere $\rho = 2\cos\phi$ that lies above the cone $\phi = \frac{\pi}{4}$.

A. $\frac{\pi}{2}$

B. $\frac{3\pi}{4}$

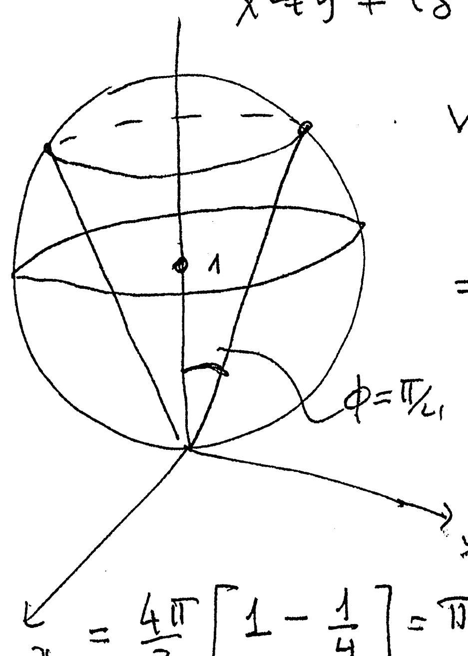
C. $\frac{3\pi}{2}$

D. $\frac{5\pi}{4}$

E. $\boxed{\pi}$

$$\rho^2 = 2\rho\cos\phi, \quad \rho^2 = 2z; \quad x^2+y^2+z^2 = 2z$$

$$x^2+y^2+(z-1)^2 = 1$$



$$V = \int_0^{2\pi}\int_0^{\pi/4}\int_0^{2\cos\phi} \rho^2 \sin\phi \, d\rho\, d\phi\, d\theta$$

$$= \frac{8}{3}\int_0^{2\pi}\int_0^{\pi/4} \cos^3\phi \sin\phi \, d\phi\, d\theta$$

$$= \frac{16\pi}{3}\int_0^{\pi/4} \cos^3\phi \sin\phi \, d\phi \qquad u = \cos\phi$$

$$= \frac{16\pi}{3}\int_{1/\sqrt{2}}^{1} u^3\, du =$$

$$= \frac{4\pi}{3}\left[1 - \frac{1}{4}\right] = \pi$$

11. **(8 points)** Evaluate the line integral

$$\int_C x^3 \, ds \text{ where}$$

$$C = \{x = t, \ y = \frac{\sqrt{2}}{2} t^2, \ z = \frac{t^3}{3}, \ 0 \le t \le 1\}.$$

A. $\frac{2}{3}$

B. $\frac{11}{30}$

C. $\frac{14}{35}$

D. $\boxed{\frac{5}{12}}$

E. $\frac{3}{14}$

$$ds = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2} \, dt =$$

$$\sqrt{1 + 2t^2 + t^4} \, dt = \sqrt{(1+t^2)^2} \, dt = (1+t^2) \, dt$$

$$\int_C x^3 ds = \int_0^1 t^3 (1+t^2) \, dt = \int_0^1 (t^3 + t^5) \, dt$$

$$= t^4/4 + t^6/6 \Big|_0^1 = \frac{1}{4} + \frac{1}{6} = \frac{10}{24} = \frac{5}{12}$$

12. **(8 points)** Find a function $f(x, y, z)$ such that

$$\nabla f(x, y, z) = \vec{F}(x, y, z) = \langle yz, xz, xy \rangle$$

and use it to compute the line integral

$$\int_C \vec{F} \cdot d\vec{r} \text{ where}$$

$$C = \{x = t^3, \ y = 1 + t^2, \ z = (1+t)^2, \ 0 \le t \le 1\}.$$

A. $\boxed{8}$

B. 4

C. 5

D. 7

E. 3

$\frac{\partial f}{\partial x} = yz$; $f(x,y,z) = xyz$ verifies

$\frac{\partial f}{\partial y} = xz$ and $\frac{\partial f}{\partial z} = xy$ so $\vec{F} = \nabla (xyz)$

Starting point of C: $(0, 1, 1)$

End point of C: $(1, 2, 4)$

$$\int_C \vec{F} \cdot d\vec{r} = f(1,2,4) - f(0,1,1) = 8$$