**Problem of the Week**

Solution of Problem No. 1 (Fall 2001 Series)

**Problem:** Determine all the square integers whose decimal representations end in 2001. What is the smallest of these numbers?

**Solution** (by Mike Hamburg, Sr. St. Joseph H.S., South Bend)

We seek \( n^2 \equiv 2001 \pmod{10^4} \). \( n = 1001 \) is obviously a solution, so \( n^2 \equiv 1001^2, n^2 - 1001^2 \equiv 0 \), so \((n+1001)(n-1001) \equiv 0 \) (all mod \( 10^4 \)). \( 10^4 = 2^4 \cdot 5^4 \), so \( 2^4 | (n+1001)(n-1001) \). Although 2 can divide both \( n + 1001 \) and \( n - 1001 \), 4 cannot divide them both because they differ by 2002. Similarly, \( 5^4 | (n + 1001)(n - 1001) \) and since 5 cannot divide them both, \( 5^4 | (n + 1001) \) or \( 5^4 | (n - 1001) \). We also have \( 8 | (n + 1001) \) or \( 8 | (n - 1001) \). Reducing mod \( 5^4 = 625 \) and 8, we have \( n \equiv \pm 1 \pmod{8} \) and \( n \equiv \pm 249 \pmod{5^4} \). Since 625 \( \equiv 1 \pmod{8} \) and 8 is inverse to 547 (mod 625), the Chinese Remainder Theorem gives us \( n \equiv (\pm 1) \cdot 625 + (\pm 249) \cdot 8 \cdot 547 \pmod{5^4 \cdot 8 = 5000} \). Reducing mod 5000, we have \( n \equiv 249, 1001, 3999 \) or 4751 (mod 5000). We check that the squares of these numbers end in 2001 and that \((n + k5000)^2 = k^25000^2 + 10000kn + n^2 \equiv n^2 \pmod{5000} \).

Also solved (at least partially) by:

**Undergraduates:** Jim Hill (Jr. MA), Piti Irawan (Sr. CS/MA), Aftab Mohammed Jalal (So. CS/MA), Stevie Schrauder (Sr. CS/MA), Eric Tkaczyk (Jr. EE/MA)

**Graduates:** Rajender Adibhatla (MA), John Hunter (MA), Chris Lomont (MA), K. H. Sarma (Nuc E), Amit Shirsat (CS), P. Ghosh & D. Subramanian (CHME)

**Faculty & Staff:** Steven Landy (Phys. at IUPUI), Chris Maxwell (OB & FC, Purdue)

**Others:** Jonathan Landy (Warren Central H.S., Indpls), Jason VanBilliard (Fac. Phila. Biblical Univ. Langhorne, PA), Aditya Utturwar (Grad. AE, Georgia Tech)

One unacceptable solution was received.