PROBLEM OF THE WEEK
Solution of Problem No. 2 (Fall 2001 Series)

Problem: Given a line $\ell$ and points $P, Q$ in a plane with $\ell$ on opposite sides of $\ell$.
   a) Determine a point $R$ on $\ell$ which maximizes $|PR| - |QR|$.
   b) Does such a point $R$ always exist?

Solution (by Damir D. Dzhafarov, Fr. MA)

Reflect $P$ over $\ell$, denoting its image on the other side $P'$, and let $R$ be any point on $\ell$. From the triangle inequality it follows that $|PR| - |QR| = |P'R| - |QR| \leq |P'Q|$ so that the left side of the inequality is maximized when it equals the right. This occurs when the three points $P', Q$, and $R$ are collinear. Thus, constructing $R$ by producing $P'Q$ until it crosses $\ell$ maximizes $|PR - QR|$. However, if $P$ and $Q$ are equidistant from $\ell$ then $P'Q$ will be parallel to $\ell$ and it will not be possible to find such an $R$ by the above method. In this eventuality $|PR| - |QR| < |P'Q|$ with the left-hand difference getting arbitrarily close to $|P'Q|$ for distant enough $R$. Hence, no $R$ makes $|PR| - |QR|$ a maximum in this case.

Also solved by:

Undergraduates: Eric Tkaczyk (Jr. EE/MA)

Graduates: Tamer Cakici (ECE), Ashish Rao (EE), K. H. Sarma (Nuc E), D. Subramanian (CHME)

Faculty: Steven Landy (Phys. at IUPUI),

Others: Michael Hamburg (Sr. St. Joseph’s H.S., South Bend)

One unacceptable solution was received.

Three late solutions to Problem 1 were received which were at least partially correct.

Undergraduate: Shyan Jeng Ho (EE)

Graduate: Ashish Rao (EE)

Other: Dan Vanderhan (St. Joseph’s H.S., South Bend)