PROBLEM OF THE WEEK
Solution of Problem No. 4 (Fall 2001 Series)

Problem: Evaluate $S_n = \sum_{k=0}^{n} (-1)^k \binom{3n}{k}$ for $n = 1, 2, \ldots$.

Solution (by Eric Tkaczyk, Jr. EE and MA)

\[ S_n = \sum_{k=0}^{n} (-1)^k \binom{3n}{k} = \sum_{k=0}^{n} (-1)^k \left[ \binom{3n-1}{k-1} + \binom{3n-1}{k} \right] \]

(by Pascal’s triangle, defining $\binom{n}{-1} = 0, \forall n \in \mathbb{Z}$), a telescoping sum! Hence

\[ S_n = (-1)^n \binom{3n-1}{n}. \]

Also solved by:

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Others: Michael Hamburg (Sr. St. Joseph’s H.S., South Bend), Jonathan Landy (Warren Central H.S., Indpls), Rob Pratt (Gr. U.N.C., Chapel Hill, NC), Mr. Rice’s class (East Tipp. Middle Sch., Laf)