**Problem of the Week**

Solution of Problem No. 9 (Fall 2001 Series)

**Problem:** Determine, with proof, all the real-valued differentiable functions \( f \), defined for real \( x > 0 \), which satisfy \( f(x) + f(y) = f(xy) \) for all \( x, y > 0 \).

**Solution** (by Brahma N. R. Vanga, Gr. Nucl. Eng., edited by the Panel)

Differentiation w.r.t. \( x \) and then w.r.t. \( y \) gives

\[
\begin{align*}
    f'(x) &= yf'(xy), \\
    f'(y) &= xf'(x, y),
\end{align*}
\]

hence

\[
xf'(x) = yf'(y) \quad \forall x, y > 0,
\]

so

\[
x f'(x) = c \text{ (constant)}.\]

Integration gives \( f(x) = c \ln x + C \), but since \( f(1) + f(1) = f(1), f(1) = 0 \), so \( C = 0 \). The general solution is

\[
f(x) = c \ln x, \quad c \in \mathbb{R}.
\]

Also solved by:

**Undergraduates:** Damir Dzhafarov (Fr. MA), Haizhi Lin (Jr. MA), Gregg Sutton (Fr. Sci.)

**Graduates:** Danlei Chen (CHME), Keshavdas Dave (EE), Gajath Gunatillake (MA), George Hassapis (MA), Sravanthi Konduri (CE), A. Mangasuli (MA), Ashish Rao (EE), Amit Shirsat (CS), D. Subramanian & P. Ghosh (CHME)

**Faculty:** Steven Landy (Phys. at IUPUI)

**Others:** Jayprakash Chipalkatti (U.B.C. Canada), Donald Dichmann (Calif.), Jing Shao (Gr. So. China Tech.)

One unacceptable solution was received.