**Problem of the Week**  
Solution of Problem No. 10 (Fall 2001 Series)

**Problem:** Given a triangle $\triangle ABC$ and a point $S$ inside, show that, if the areas of triangles $\triangle ABS, \triangle BCS, \triangle CAS$ are equal, then $S$ is the centroid of $\triangle ABC$.

**Solution** (by Steven Landy, Fac. Physics at IUPUI, edited by the Panel)

Let $\overline{AS}, \overline{BS}, \overline{CS}$ be extended to intersect $\overline{BC}, \overline{AC}, \overline{AB}$ in $A', B', C'$, resp. Let $\langle ABC \rangle$ denote the area of $\triangle ABC$, similarly for other triangles. Let $q$ denote the common area of $\triangle ASB, \triangle BSC, \triangle CSA$. Now

$$\frac{\langle ASC'' \rangle}{\langle BSC'' \rangle} = \frac{|AC'|}{|BC'|}, \text{ the triangles have the same height}$$

$$\frac{\langle ACC'' \rangle}{\langle BCC'' \rangle} = \frac{|AC'|}{|BC'|}, \text{ the triangles have the same height.}$$

So

$$\frac{\langle ACC'' \rangle}{\langle BCC'' \rangle} = \frac{\langle ASC'' \rangle + q}{\langle BSC'' \rangle + q} = \frac{\langle ASC'' \rangle}{\langle BSC'' \rangle},$$

$$1 + \frac{q}{\langle BSC'' \rangle} = \frac{\langle BSC'' \rangle}{\langle BSC'' \rangle} = \frac{\langle ASC'' \rangle + q}{\langle ASC'' \rangle} = 1 + \frac{q}{\langle ASC'' \rangle}$$

which implies $\langle ASC'' \rangle = \langle BSC'' \rangle$, then $|AC'| = |BC'|$, so $\overline{CC'}$ is a median of $\triangle ABC$. So are $\overline{AA'}, \overline{BB'}$, $S$ is the intersection of the medians, $S$ is the centroid.

Also solved by:

**Undergraduates:** Stevie Schraudner (Sr. CS/MA), Eric Tkaczyk (Jr. EE/MA)

**Graduates:** Ali R. Butt (ECE), Keshavdas Dave (EE), Gajath Gunatillake (MA), George Hassapis (MA), Ashish Rao (EE), Amit Shirsat (CS), D. Subramanian & P. Ghosh (CHME)

**Others:** Dane Brooke, Jayprakash Chipalkatti (U.B.C Canada), Michael Hamburg (Sr. St. Joseph’s H.S., South Bend), Kunarajasingam Jeevarajan (Sri Lanka), Jonathan Landy (Warren Central H.S., Indpls)

Two unacceptable solutions were received.