PROBLEM OF THE WEEK
Solution of Problem No. 12 (Fall 2001 Series)

Problem: Evaluate $\int_0^\pi \frac{\cos 4x - \cos 4\alpha}{\cos x - \cos \alpha} \, dx$.

Solution (by the Panel)

$$\cos 4x - \cos 4\alpha = 2\cos^2 2x - 1 - 2\cos^2 2\alpha + 1$$
$$= 2(\cos 2x + \cos 2\alpha)(\cos 2x - \cos 2\alpha)$$
$$= 4(\cos 2x + \cos 2\alpha)(\cos x + \cos \alpha)(\cos x - \cos \alpha).$$

$$I = \int_0^\pi \frac{\cos 4x - \cos 4\alpha}{\cos x - \cos \alpha} \, dx = 4 \int_0^\pi (\cos 2x + \cos 2\alpha)(\cos x + \cos \alpha) \, dx$$
$$= 4 \int_0^\pi (\cos 2x \cos x + \cos 2x \cos \alpha + \cos 2\alpha \cos x + \cos 2\alpha \cos \alpha) \, dx$$
$$= 4 \int_0^\pi \left(\frac{1}{2}(\cos 3x + \cos x) + \cos 2x \cos \alpha + \cos 2\alpha \cos x + \cos 2\alpha \cos \alpha\right) \, dx.$$ 

Since

$$\int_0^\pi \cos kx \, dx = 0 \quad (k = 1, 2, 3, \ldots),$$

consequently

$$I = 4 \cos 2\alpha \cos \alpha.$$

Solved by:

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One incorrect solution was received. We received a correct late solution of Problem 11 from Rob Pratt (U. North Carolina).