PROBLEM OF THE WEEK
Solution of Problem No. 1 (Fall 2002 Series)

Problem: Suppose \( f(x) \) and \( g(x) \) are polynomials of degrees \( m > n > 0 \), respectively. Write \( \frac{f(x)}{g(x)} = q(x) + \frac{r(x)}{g(x)} \), where \( q(x) \) and \( r(x) \) are polynomials and the degree of \( r(x) \) is less than the degree of \( g(x) \). Let \( S(h) \) denote the sum of the zeros of a polynomial \( h(x) \). Show that \( S(q) = S(f) - S(g) \).

Solution (by Chris Lomont, graduate (MA), edited by the Panel)

Given is

\[
(*) \quad f = gq + r,
\]

where \( \deg f = m, \quad \deg g = n < m, \quad \deg r < n \).

WLOG may assume leading coefficients of \( f \) and \( g \) are 1. A well known result of elementary algebra is that if \( f(x) = x^m + a_1 x^{m-1} + \ldots \), then \( a_1 = -S(f) \). So comparing the coefficients of \( x^{m-1} \) in (*):

\[
-S(f) = -S(gq) = -S(g) - S(q),
\]

i.e. \( S(q) = S(f) - S(g) \).

Also solved by:

Undergraduates: Jason Anderson (Fr. ME), Eric Tkaczyk (Sr. MA/EE)

Graduates: Parsa Bakhtary (MA), Prasenjeet Ghosh (ChE), Ashwin Kumar (ME), Ashish Rao (ECE), K. H. Sarma (Nuc), Amit Shirsat (CS), Jasvinder Singh (ECE), Melissa Wilson (MA), Thierry Zell (MA)

Faculty: Steven Landy (Physics at IUPUI)

Others: J.L.C. (Fishers, IN), Jonathan Landy (Fr., Cal Tech), M.L.R. (Iowa St. U.)

Four unacceptable solutions were received.