PROBLEM OF THE WEEK

Solution of Problem No. 1 (Fall 2002 Series)

Problem: Suppose $f(x)$ and $g(x)$ are polynomials of degrees $m > n > 0$, respectively. Write $\frac{f(x)}{g(x)} = q(x) + \frac{r(x)}{g(x)}$, where $q(x)$ and $r(x)$ are polynomials and the degree of $r(x)$ is less than the degree of $g(x)$. Let $S(h)$ denote the sum of the zeros of a polynomial $h(x)$. Show that $S(q) = S(f) - S(g)$.

Solution (by Chris Lomont, graduate (MA), edited by the Panel)

Given is

$$(*) \qquad f = gq + r,$$

where $\deg f = m, \quad \deg g = n < m, \quad \deg r < n$.

WLOG may assume leading coefficients of f and g are 1. A well known result of elementary algebra is that if $f(x) = x^m + a_1 x^{m-1} + \ldots$, then $a_1 = -S(f)$. So comparing the coefficients of x^{m-1} in $(*)$:

$$-S(f) = -S(gq) = -S(g) - S(q),$$

$$\text{i.e.} \qquad S(q) = S(f) - S(g).$$

Also solved by:

Undergraduates: Jason Anderson (Fr. ME), Eric Tkaczyk (Sr. MA/EE)

Graduates: Parsa Bakhtary (MA), Prasenjeet Ghosh (ChE), Ashwin Kumar (ME), Ashish Rao (ECE), K. H. Sarma (Nuc), Amit Shirsat (CS), Jasvinder Singh (ECE), Melissa Wilson (MA), Thierry Zell (MA)

Faculty: Steven Landy (Physics at IUPUI)

Others: J.L.C. (Fishers, IN), Jonathan Landy (Fr., Cal Tech), M.L.R. (Iowa St. U.)

Four unacceptable solutions were received.