PROBLEM OF THE WEEK
Solution of Problem No. 8 (Fall 2002 Series)

Problem: Show that $a = \sqrt[3]{7+5\sqrt{2}} + \sqrt[3]{7-5\sqrt{2}}$ is an integer. (Calculator or computer solutions are not acceptable.)

Solution (by E. Tkaczyk, Sr. EE and MA)

Note that the expression clearly yields a real result for a. Straightforward arithmetic shows that this real a must satisfy $a^3 = 14 - 3a$, or $(a-2)(a^2+2a+7) = 0$. As the quadratic factor has no real solutions ($2^2 - 4(1)(7) < 0$), 2 is clearly the only real root of the equation. Thus, $a = 2$, an integer.

Also solved by:

Undergraduates: Yu Wei Lu (Jr. EE), Ryan Machtmes (Sr. E&AS), Robert Moore (Fr. E), Ben Niehoff (Jr. ECE)

Graduates: Dionysios Aliprantis (ECE), N. V. Krishna (CS), Jia-Han Li (ECE), Chris Lomont (MA), K. H. Sarma (NucE), YiHuang Shen (MA), Qi Xu (ChE)

Faculty: Steven Landy (Physics at IUPUI)

Others: J.L.C. (Fishers, IN), Guillermo Fornerod (Argentina), Jonathan Landy (Fr., Cal Tech), Vijay Madhavapeddi (Newark, CA), Kishin Sadarangani & Luis Gonzales Sanchez (MA, Univ of Tafira, Las Palmas, Canaries), Dharmashankar Subramanian (Honeywell Labs, Minneapolis, MN), Unknown (Fr at UCSD, LaJolla)

Several late solutions to Problem 7 were received: Ryan Machtmes, Ben Niehoff, Eric Tkaczyk, Dionysios Aliprantis, Rob Pratt, Dharmashankar Subramanian, were correct.

Two incorrect late solutions of Problem 7 were received.