PROBLEM OF THE WEEK
Solution of Problem No. 10 (Fall 2002 Series)

**Problem:** Given a triangle $T$ with points $A, B, C$, one on the interior of each side, let $\Gamma$ be the circle passing through $A, B$ and $C$. Show that $\Gamma$ is not smaller than the incircle of $T$.

**Solution** (by Yifau Liang, Gr. ECE)

Let $d_a, d_b, d_c$ denote the distances of the center $O$ of $\Gamma$ from the sides $a, b, c$ resp., let $r$ denote the radius of $\Gamma$, $\rho$ the radius of the incircle. Clearly,

$$d_a, d_b, d_c \leq r.$$  

The area of $T$ is given by

$$|T| = \frac{1}{2}(ad_a + bd_b + cd_c) \leq \frac{1}{2}(a + b + c)r$$

if $O$ is inside $T$. Otherwise, there are one or two minus signs in the first sum but the upper bound remains the same. But also

$$|T| = \frac{1}{2}(a + b + c)\rho.$$  

Hence $\rho \leq r$.

Also solved by:

**Faculty:** Steven Landy (Physics at IUPUI)

Correct late solutions were received from Eric Tkaczyk (Sr. EE/MA) and George Hassapis (Gr. MA)

One incorrect late solution was received.