**Problem of the Week**
Solution of Problem No. 10 (Fall 2005 Series)

**Problem:** Which positive integers $n$ are expressible in at least one way as the sum of two or more consecutive positive integers? Prove your answer.

**Solution** (by Prithwijit De & Sonali Dasgupta, U.C.C, Republic of Ireland)

Claim: All positive integers $n$ except the powers of 2 can be written as sum of two or more consecutive positive integers. Suppose $n$ can be written as a sum of $\ell$ consecutive numbers beginning with $(k+1)$. Then

$$n = (k+1) + (k+2) + \ldots + (k+\ell) = \frac{\ell(2k+\ell+1)}{2}.$$

Now, one of $\ell$ or $(2k+\ell+1)$ is odd (and the other one is even). Therefore, $n$ is not a power of 2.

Conversely, let $n$ be a positive integer with an odd factor. Since $n$ has an odd factor, so does $2n$, and we can write $2n = f_1f_2$ where one of $f_1$ or $f_2$ is odd, the other one is even, and $1 < f_1 < f_2$. Let $k = \frac{f_2-f_1-1}{2}$, $\ell = f_1$, then $f_2 = 2k + \ell + 1$, so that

$$n = \frac{f_1f_2}{2} = \frac{\ell(2k+\ell+1)}{2} = (k+1) + (k+2) + (k+3) + \ldots + (k+\ell).$$

At least partially solved by:

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