Problem No. 9 (Fall 2006 Series)

Let $E$ be an ellipse with area 1. Given are two chords, parallel to the major and minor axes, respectively, that divide $E$ into four regions. Prove that at least two of those regions have area not exceeding $\frac{1}{4}$.