Problem: Show that \( \sum_{k=1}^{n} \frac{1}{k} \left( \binom{n}{k} + 1 \right) = \sum_{k=1}^{n} \frac{2^k}{k} \).

This problem is identical to Problem #10 from Fall 2006. The panel apologizes for the duplication and thanks Steve Spindler and Nate Orlow for pointing it out. This problem will not be counted for the semester’s competition. The solution provided below is different from the previously published one.

Solution (by Noah Blach, Freshman, Math)

Let \( U_n = \sum_{k=1}^{n} \left( \binom{n}{k} + 1 \right) \frac{1}{k} \)

\[
U_{n+1} - U_n = \sum_{k=1}^{n+1} \left( \binom{n+1}{k} + 1 \right) \frac{1}{k} - \sum_{k=1}^{n} \left( \binom{n}{k} + 1 \right) \frac{1}{k}
\]

\[
= \frac{2}{n+1} + \sum_{k=1}^{n} \left( \binom{n+1}{k} - \binom{n}{k} \right) \frac{1}{k} = \frac{2}{n+1} + \sum_{k=1}^{n} \binom{n}{k-1} \frac{1}{k}
\]

\[
= \frac{2}{n+1} + \sum_{k=1}^{n} \frac{n!}{(k-1)! \cdot k \cdot (n-k+1)!} = \frac{2}{n+1} + \sum_{k=1}^{n} \frac{(n+1)!}{k! \cdot (n-k+1)! \cdot n+1}
\]

\[
= \frac{2}{n+1} + \frac{1}{n+1} \sum_{k=1}^{n} \binom{n+1}{k} = \frac{2}{n+1} + \frac{1}{n+1} \left( 2^{n+1} - 2 \right) = \frac{2^{n+1}}{n+1}
\]

\[ U_1 = \frac{2}{1} = 2^1, \quad \text{and} \quad U_n = U_1 + \sum_{k=2}^{n} \left( U_k - U_{k-1} \right) = U_1 + \sum_{k=2}^{n} \frac{2^k}{k} \]

\[ = \sum_{k=1}^{n} \frac{2^k}{k}. \]

Also solved by:

Undergraduates: Lokesh Batra (Fr. Engr), Ankit Kuwadekar (Fr. CS), Hetong Li (Fr. Science), Siddharth Tekriwal (So. Engr.) Fan Zhang (So. CS)
Graduates: Tom Engelsman (ECE)

Others: Manuel Barbero (New York), Brian Bradie (Christopher Newport U. VA), Mihaela Dobrescu (Faculty, Christopher Newport Univ.), Hoan Duong (San Antonio College), Elie Ghosn (Montreal, Quebec), Steven Landy (IUPUI Physics), Thomas Murray (Sr. UW-Medison), Sorin Rubinstein (TAU faculty, Israel) Kishin K. Sadarangani (Professor, ULPGC, Spain), Steve Spindler (Chicago)