Problem: Show that if \( m, n \) are positive integers then the smaller of \( \sqrt[3]{m} \) and \( \sqrt[3]{n} \) is no larger than \( \sqrt[3]{3} \).

Solution (by Huanyu Shao, Graduate student, Computer Science, Purdue University)

Assume \( m \leq n \). Then \( \frac{1}{m} \geq \frac{1}{n} > 0 \) then \( m^{\frac{1}{m}} \leq m^{\frac{1}{n}} \) (because \( m \) is a positive integer). So, the smaller of \( m^{\frac{1}{m}}, n^{\frac{1}{n}} \) is no larger than the larger of \( m^{\frac{1}{m}} \) and \( n^{\frac{1}{n}} \).

We then try to prove that \( \max \limits_{m \in \mathbb{N}} m^{\frac{1}{m}} = \sqrt[3]{3} \).

Let

\[
f(x) = x^{\frac{1}{x}} \quad (x > 0)
\]

\[
f'(x) = (e^{\ln x})' = e^{\ln x} \cdot x^{-1} \cdot \frac{\frac{1}{x} \cdot x - \ln x}{x^2} = x^{\frac{1}{x}} \cdot 1 - \ln x
\]

So \( f'(x) > 0 \) when \( x < e \), \( f'(x) < 0 \) when \( x > e \). So \( f \) decreases when \( x > e \).

\( m^{\frac{1}{m}} \) decreases when \( n \geq 3 \). And we also have \( \sqrt[3]{1} < \sqrt[3]{2} < \sqrt[3]{3} \).

So \( \max \limits_{m \in \mathbb{N}} m^{\frac{1}{m}} = \sqrt[3]{3} \).

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