PROBLEM OF THE WEEK
Solution of Problem No. 4 (Fall 2008 Series)

**Problem:** Let \( f \) be a real–valued function on \([0, \infty]\) such that
\[
\lim_{x \to \infty} f(x) = \lim_{x \to \infty} f'''(x) = 0.
\]
Show that \( \lim_{x \to \infty} f'(x) = \lim_{x \to \infty} f''(x) = 0 \).

**Solution** (by Minghua Lin & Zhang Xiao, Shaanxi Normal University, China)

\( \forall x \in [0, +\infty) \), we have
\[
f(x + 1) = f(x) + f'(x) + \frac{f'''(x)}{2!} + \frac{f'''(\xi)}{3!}, \quad \xi \in (x, x + 1) \quad (1)
f(x + 2) = f(x) + 2f'(x) + 2f''(x) + \frac{8f'''(\eta)}{3!}, \quad \eta \in (x, x + 2) \quad (2)
\]
Let \( x \to +\infty \), then \( \xi \to +\infty \) and \( \eta \to +\infty \).
From (1), we have \( \lim_{x \to +\infty} \left[ f'(x) + \frac{f''(x)}{2} \right] = 0 \) \( (3) \)
From (2), we have \( \lim_{x \to +\infty} [f'(x) + f''(x)] = 0 \) \( (4) \)
From (3) – (4), we have \( \lim_{x \to +\infty} f''(x) = 0 \) and so \( \lim_{x \to +\infty} f'(x) = 0 \).

Also solved by:

**Graduates:** Britain Cox (Math)

**Others:** Hoan Duong (San Antonio College), Elie Ghosn (Montreal, Quebec), Steven Landy (IUPUI Physics staff), Sorin Rubinstein (TAU faculty, Israel)