**Problem:** Let $C$ be a closed convex curve with continuously turning tangent. Prove that, if $\triangle$ is an inscribed triangle of maximal perimeter, then the normals to $C$ at the vertices of $\triangle$ bisect the angles of $\triangle$.

**Solution** (by Andrea Altamura, Graduate student, Italy)

Let $P_0$, $P_1$ and $P_2$ the vertices of the triangle of maximal perimeter inscribed in $C$. Without loss of generality we can assume that $P_0 = (0,0)$ and $P_1 = (a,0)$. Let

$$C = \{(x(t), y(t)) : t \in [0,1]\}, \quad P_t = (x(t), y(t)).$$

Consider $p(t)$ the perimeter of the $\triangle$ determined by $P_0$, $P_1$ and $P_t$, that is

$$p(t) = |P_t - P_0| + |P_t - P_1| + |P_1 - P_0| = \sqrt{x(t)^2 + y(t)^2} + \sqrt{(x(t) - a)^2 + y(t)^2} + |a|$$

for $t \in [0,1]$. Finding the critical points, we get

$$0 = p'(t)$$

$$= \frac{x(t)x'(t) + y(t)y'(t)}{\sqrt{x(t)^2 + y(t)^2}} + \frac{(x(t) - a)x'(t) + y(t)y'(t)}{\sqrt{(x(t) - a)^2 + y(t)^2}}$$

$$= \frac{\langle x(t), y(t) \rangle \cdot \langle x'(t), y'(t) \rangle}{|P_t - P_0|} + \frac{\langle x(t) - a, y(t) \rangle \cdot \langle x'(t), y'(t) \rangle}{|P_t - P_1|}$$

$$= |\langle x'(t), y'(t) \rangle| \cos(\theta_0) - |\langle x'(t), y'(t) \rangle| \cos(\theta_1)$$

where $\theta_0$ is the angle between the vector $P_t - P_0$ and the tangent line of $C$ at $P_t$, and $\theta_1$ is the angle between the $P_t - P_1$ and the tangent line of $C$ at $P_t$ (with opposite direction). Since the curve has continuously turning tangent we can choose the parametrization such that $|\langle x'(t), y'(t) \rangle| \neq 0$. Thus $\theta_0 = \theta_1$. Now, since $C$ is convex, this is the same that saying that at all critical points of $p(t)$ the normal of $C$ at $P_t$ bisects the angles of the $\triangle(P_0, P_1, P_t)$ at $P_t$. Since $P_2$ is a critical point of $p(t)$ the proof is finished.

The problem was also solved by:

**Graduates:** Vitezslav Kala (Math), Tairan Yuwen (Chemistry)

**Others:** Mohamed Alimi (Tunisia), Manuel Barbero (New York), Gruian Cornel (IT, Romania), Elie Ghosn (Montreal, Quebec), Steven Landy (IUPUI Physics staff), Sorin Rubinstein (TAU faculty, Israel), Craig Schroeder (Grad student, Stanford Univ.)