Problem: Let $A$ be any set of 39 distinct integers chosen from the arithmetic progression $6, 33, 60, \ldots, 1977$.

Prove that there must be two distinct integers in $A$ whose sum is 2010.

Solution (by Eric Haengel, Junior, Physics/Math)

The numbers $6, 33, 60, \ldots, 1977$ are all of the form $6 + 27n$ where $n \in A = \{0, \ldots, 73\}$. If $n, m \in A$ such that $n + m = 74$, then

$$(6 + 27n) + (6 + 27m) = 12 + 27 \cdot 74 = 2010.$$ 

So it suffices to show that any collection of 39 distinct integers in $A$ will contain two numbers that add up to 74. Suppose the contrary: there exists a subset $B$ of $A$ containing 39 integers, such that no two add up to 74.

Thus, if $n \in B$, $(74 - n) \notin B$. This means that $B$ cannot contain both numbers in the pairs $(1, 73), (2, 72), \ldots, (36, 38)$. Apart from these, $B$ may contain 0 and 36, and counting it all up, $B$ can contain at most $1 + 1 + 36 = 38$ elements, which is a contradiction.

The problem was also solved by:

Undergraduates: William A. Arnold (Sr. Science), Ka Wang Chow (Sr. Science) Kilian Cooley (So.), David Elden (Sr. Mech. Engr), Robb Glasser (So. CS & Math), Robert Gustafson (Sr. CS), Han Liu (Fr. Math), Michael Monte (Fr. Engr.), Jorge Ramos, Anurag Somani (Fr. Phys)

Graduates: Sonia Belaid (ECE), Jyotishka Datta (Stat.), Rodrigo Ferraz de Andrade (Math), Benjamin Philabaum (Phys.), Huanyu Shao (CS), Arnold Yim (Math), Tairan Yuwen (Chemistry)

Others: Neacsu Adrian (Romania), Syd Amit (Graduate student, Boston College), Tomer Amit (Faculty, TAU, Israel), Manuel Barbero (New York), Brendan Berger (Jr. high school student, MD), Gruian Cornel (IT, Romania), Sandipan Dey (Graduate student, UMBC), Tom Engelsman (Chicago, IL), Elie Ghosn (Montreal, Quebec), Jeffery Hein (CS & Math,
Purdue Univ. Calumet), Steven Landy (IUPUI Physics staff), Kevin Laster (Indianapolis, IN), Wei-hsiang Lien (Research assistant, National Chiao-Tung Univ., Taiwan), Louis Rogliano (Corsica), Craig Schroeder (Ph.D. student, Stanford Univ.), Mark Sellke (Harrison High School, IN), Steve Spindler (Chicago), Sahana Vasudevan (9th grade, Palo Alto HS, San Jose, CA), Turkay Yolcu (Visiting at Purdue U.)