**Problem:** Given nine lattice points in space, show that there is an interior lattice point on at least one segment joining a pair of them.

Note: A “lattice point” is a point whose $x$-, $y$-, and $z$-coordinates are all integers.

**Solution:** (by Kaibo Gong, Senior, Mathematics, Purdue University)

Consider function $\phi(a_1, a_2, a_3) = (a_1 \mod 2, a_2 \mod 2, a_3 \mod 2)$ ($a_1, a_2, a_3 \in \mathbb{Z}$). Thus this function has 8 results. $(0, 0, 0)(0, 0, 1)(0, 1, 0)(1, 0, 0)(1, 0, 1)(1, 1, 0)$ and $(1, 1, 1)$. (Here 0 means $a_i$ is even and 1 means odd.)

Thus for the 9 lattice points in space from pigeon–hole thm, at least two points will get the same result. Say, $\psi(m) = \psi(n)$ $m = (m_1, m_2, m_3)$ $n = (n_1, n_2, n_3)m_i \in \mathbb{Z}, n_i \in \mathbb{Z}$. Thus $m_1 \equiv n_1 \mod 2$ $m_2 \equiv n_2 \mod 2$ $m_3 \equiv n_3 \mod 2$ which means $m_1 - n_1 = 2k_1$, $m_2 - n_2 = 2k_2$, $m_3 - n_3 = 2k_3$, $k_1, k_2, k_3 \in \mathbb{Z}$. Thus, the point $k = (k_1, k_2, k_3)$ is the mid-point of the line segment $mn$.

Thus $k$, the mid–point of $mn$ is a lattice point.

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