PROBLEM OF THE WEEK
Solution of Problem No. 7 (Fall 2011 Series)

**Problem:** For every integer \( n \geq 2 \), prove that

\[
\sum_{k=1}^{n} (-1)^k k \binom{n}{k} = 0,
\]

where \( \binom{n}{k} \) is the usual binomial coefficient.

**Solution:** (by Hubert Desprez, Paris, France)

Let’s consider \( \varphi(x) = \sum_{q=0}^{n} \binom{n}{q} x^q = (1 + x)^n \). We have

\[
\varphi'(x) = \sum_{q=0}^{n} q \binom{n}{q} x^{q-1} = n(1 + x)^{n-1},
\]

which implies

\[
-\sum_{q=0}^{n} q \binom{n}{q} (-1)^{q-1} = -\varphi'(-1) = 0.
\]

The problem was also solved by:

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