Problem No. 2 (Fall 2013 Series)

Let $f$ be nonnegative, continuous, and strictly increasing on $[0,1]$. For $p > 0$, let $x_p$ be the number in $(0,1)$ which satisfies

$$f^p(x_p) = \int_0^1 f^p(x) \, dx.$$

Find $\lim_{p \to \infty} x_p$. 