Problem:
Let \( f \) be nonnegative, continuous, and strictly increasing on \([0, 1]\). For \( p > 0 \), let \( x_p \) be the number in \((0, 1)\) which satisfies

\[
f^p(x_p) = \int_0^1 f^p(x) \, dx.
\]

Find \( \lim_{p \to \infty} x_p \).

Solution: (by Samson Zhou, Graduate student, CS, Purdue University)

Since \( f \) is nonnegative, continuous, and strictly increasing on \([0, 1]\), then for any \( y \in (0, 1) \),

\[
\int_0^1 f^p(x) \, dx \geq \int_y^1 f^p(x) \, dx
\]

\[
\int_0^1 f^p(x) \, dx \geq \int_y^1 f^p(y) \, dx
\]

\[
\int_0^1 f^p(x) \, dx \geq (1 - y) f^p(y)
\]

Hence, for any \( 0 < a < b < 1 \), \( f(a) < f(b) \), so there exists \( N \) such that for all \( n > N \),

\[
f^n(a) < (1 - b) f^n(b).
\]

That is, for any \( a \in (0, 1) \), there exists \( N \) such that for all \( n > N \),

\[
x_n > a.
\]

However, \( x_p < 1 \) for all \( p \), so by the Squeeze Theorem,

\[
\lim_{p \to \infty} x_p = 1.
\]

The problem was also solved by:

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