Problem:
Let \( a, b > 0 \). Find the curve in the first quadrant which passes through \((a, b)\) and has the property that if the tangent line is drawn at any point \( p \) on the curve then that part of this tangent line which lies in the first quadrant is bisected at \( p \).

**This problem was proposed by Tom Engelsman C.I.T. Tampa, FL.

Solution: (by Bennett Marsh, Junior, Physics/Math, Purdue University)

Let \((x, y)\) be in the first quadrant. If we were to draw a line through this point such that the portion of the line in the first quadrant was bisected at \((x, y)\), then the \(x\)- and \(y\)-intercepts of the line would have to be \(x_{int} = 2x\) and \(y_{int} = 2y\). The slope of this line is then just \(m = -y_{int}/x_{int} = -y/x\). This means that the desired curve must satisfy the differential equation

\[
\frac{dy}{dx} = -\frac{y}{x}.
\]

Integrating, this leads to

\[
y = \frac{c}{x},
\]

and plugging in the condition \(y(a) = b\), we find

\[
y = \frac{ab}{x}.
\]

The problem was also solved by:

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