Problem No. 2 (Spring 2001 Series)

Given a triangle $ABC$, choose $A_1, B_1, C_1$ on the sides opposite $A, B, C$ respectively so that the centroid of $A_1B_1C_1$ coincides with that of $ABC$. Determine (with proof) the locations of $A_1, B_1, C_1$ so that the ratio of the area of $A_1B_1C_1$ to that of $ABC$ is minimal.