Problem No. 13 (Spring 2001 Series)

Let $p$ be a prime number and let $J$ be the set of all $2 \times 2$ matrices,
\[
\begin{pmatrix}
a & b \\
c & d
\end{pmatrix}
\]
where $a, b, c, d \in \{0, 1, \ldots, p - 1\}$, and which satisfy
\[
a + b \equiv 1 \pmod{p}
\]
and
\[
ad - bc \equiv 0 \pmod{p}.
\]
How many matrices are in $J$?