Problem: Let $C$ be a smooth closed curve (no corners) in the plane with a convex interior, and $P$ a given point on $C$. Show that there are points $Q, R$ on $C$ such that $\triangle PQR$ is equilateral.

Solution (by Julien Santini, Lacordaire H.S., France; edited by the Panel)

Let an angle of $60^\circ$ revolve counter-clockwise about $P$, with initial position of one of the arms tangent to $C$ at $P$. The intercepts of the two arms are initially 0 and some $q > 0$. Turn the angle until the other arm becomes tangent to $C$, and the intercepts are now some $r > 0$ and 0. Hence the difference of the intercepts changes from $0 - q < 0$ to $r - 0 > 0$. By continuity there is a position of the two arms $\overrightarrow{PQ}, \overrightarrow{PR}$ where $|PQ| = |PR|$, hence $\triangle PQR$ is equilateral.

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