PROBLEM OF THE WEEK

Solution of Problem No. 4 (Spring 2002 Series)

Problem: Let $h(t)$ denote the point on the hyperbola H whose cartesian coordinates are $x = \cosh\, t,\ y = \sinh\, t$. Let $Q(H)$ be the set of rational points on H (i.e. both x and y are rational numbers).

a) Show that if $h(t_1)$, and $h(t_2)$ are in $Q(H)$, then so are $h(t_1 \pm t_2)$.

b) Show that if $t = \cosh^{-1}\frac{13}{12}$, then $h(kt) \in Q(H)$ for every integer k.

Solution (by Fabio Augusto Milner, Fac. Math, edited by the Panel)

First note that $h(t) \in Q(H)$ if and only if $e^t = \frac{1}{2}(e^t + e^{-t}) + \frac{1}{2}(e^t - e^{-t}) = x + y$ is in Q^+ (the class of postive rational numbers). If $h(t_1)$ and $h(t_2)$ are in $Q(H)$ then e^{t_1} and e^{t_2} are rational and so are $e^{t_1 \pm t_2} = e^{t_1} \cdot e^{\pm t_2}$, hence $h(t_1 \pm t_2) \in Q(H)$. If $\cosh\, t = \frac{13}{12}$ then $\sinh\, t = \pm\left(1 - \left(\frac{13}{12}\right)^2\right)^{\frac{1}{2}} = \pm\frac{5}{12}$, hence $e^t \in Q^+$ and $e^{kt} = (e^t)^k \in Q^+$ for every integer k.

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