**Problem of the Week**
Solution of Problem No. 7 (Spring 2003 Series)

**Problem:** Prove that every planar compact set of at least two points has a circumscribed square. (A set is compact if it is bounded and contains all its limit points. Set $S$ is circumscribed by square $Q$ if $S \subseteq Q$ and every side of $Q$ contains at least one point of $S$.)

**Solution** (by Steven Landy, staff Physics at IUPUI)

For every angle $\theta$ from a fixed reference line $\ell$ there are two supporting lines of $Q$ making angle $\theta$ with $\ell$, and two other supporting lines making angle $\theta + \frac{\pi}{2}$ with $\ell$. Let $f(\theta)$ denote the distance between the supporting lines for angle $\theta$. Define $g(\theta) = f(\theta) - f(\theta + \frac{\pi}{2})$; then $g$ is a continuous function and it changes from a value $d$ at $\theta$ to $-d$ at $\theta + \frac{\pi}{2}$. By the Intermediate Value Theorem $g = 0$ at some $\theta_x$ between $\theta$ and $\theta + \frac{\pi}{2}$. The supporting lines at $\theta_x$ and $\theta_x + \frac{\pi}{2}$ from a circumscribed square of $Q$.

Also solved by:

**Graduates:** Thierry Zell (MA)

**Others:** Regis J. Serinko (PhD, State Coll., PA)

One incorrect solution was received.