Problem: Show that $x = \tan 18^\circ$ satisfies $5x^4 - 10x^2 + 1 = 0$.

Solution (by Prasenjeet Ghosh, New Delhi, India; former Purdue student)

Let $y = 18^\circ$. Thus $5y = 90^\circ$. Now

(1) $\tan(5y) = \frac{\tan(2y) + \tan(3y)}{1 - \tan(2y)\tan(3y)}$.

Since $\tan 5y = \tan 90^\circ = \infty$, it follows that

(2) $1 - \tan(2y)\tan(3y) = 0$.

Substituting for $\tan(2y)$ and $\tan(3y)$ in Equation (2) we get

(3) $\left(\frac{2\tan(y)}{1 - \tan^2 y}\right)\left(\frac{3\tan(y) - \tan^3 y}{1 - 3\tan^2 y}\right) = 1$.

Rearranging the algebra in Equation (3), we get

(4) $5\tan^4 y - 10\tan^2 y + 1 = 0$.

Thus $x = \tan y$ satisfies Equation (4) which is the required proof.

Also solved by:

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Two unacceptable solutions were received.