Problem: Let $D_n$ be the region below the hyperbola $y = 1/x$ for $1 \leq x \leq n$ and above the union of the rectangles with base $k \leq x \leq k+1$ and height $2/(2k+3)$ for $k = 1, \cdots, n-1$. Determine $\lim_{n \to \infty} \text{(area of } D_n\text{)}$.

Solution (by the Panel)

$$D_n = \log n - 2\sum_{k=1}^{n-1} \frac{1}{2k+3},$$

where

$$2\sum_{k=1}^{n-1} \frac{1}{2k+3} = \sum_{k=1}^{n} \frac{1}{k} - (2\sum_{k=1}^{n} \frac{1}{2k} - 2\sum_{k=1}^{n} \frac{1}{2k+1}) - \frac{2}{3}$$

$$= \sum_{k=1}^{n} \frac{1}{k} + 2\sum_{k=1}^{2n+1} (-1)^{k-1} \frac{1}{k} - \frac{8}{3}.$$

Hence

$$\lim_{n \to \infty} D_n = \lim (\log n - \sum_{k=1}^{n} \frac{1}{k}) - 2\lim (1 - \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{2n+1}) + \frac{8}{3}$$

$$= \gamma + \frac{8}{3} - 2\log 2,$$

where $\gamma$ is Euler’s constant.

Also solved by:

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Graduates: Jianguang Guo (Phys)

Faculty: Steven Landy (Phys, IUPUI)

Others: Georges Ghosn (Quebec), Jonathan Landy (Cal. Tech.)

Three unacceptable solutions were received.

We received late solutions of Problem 2 from: Jignesh Vidyut Mehta (Jr. Phys) and Sandeep Nandy (So. Eng.)