PROBLEM OF THE WEEK
Solution of Problem No. 9 (Spring 2004 Series)

Problem: Let A, B, C, D, E be the vertices, in order, of a pentagon. Show that the pentagon has a circumscribed circle if and only if $\angle EAB + \angle ECB = \angle EAB + \angle EDB = 180°$.

Solution (by Mordechai Martin Rappaport, Staff, Worcester Yeshiva Acad.)

1. If A, B, C, D, E lie on a circle then: $\angle EAB + \angle ECB = \angle EAB + \angle EDB = 180$.

$ABCE$ is a quad with vertices on a circumscribed circle, and so $\angle EAB + \angle ECB = 180$. $ABDE$ is a quad with vertices on the same circumscribed circle, and so $\angle EAB + \angle EDB = 180$. Hence $\angle EAB + \angle ECB = \angle EAB + \angle EDB = 180$.

2. If $\angle EAB + \angle ECB = \angle EAB + \angle EDB = 180$ then A, B, C, D, E lie on a circle.

Because $\angle EAB + \angle ECB = 180$, quad $ABCE$ can be circumscribed by a circle. Because $\angle EAB + \angle EDB = 180$, quad $ABDE$ can be circumscribed by a circle. Triangle EAB is circumscribed by both circles, and so they must be identical, because there is only one circle that a triangle can be circumscribed by.

Also solved by:

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A late, correct, solution to Problem 8 was received from M. A. Padon and A. Plata (U.L.P.G.C. Spain)