Problem No. 8 (Spring 2005 Series)

Let $p_1, p_2, \ldots, p_n$ be points on a sphere of radius 1 in $\mathbb{R}^3$. Let $d_{ij}$ be the distance (in $\mathbb{R}^3$) from $p_i$ to $p_j$.

(a) Prove that $\sum_{i<j} d_{ij}^2 \leq n^2$.

(b) When is $\sum d_{ij}^2 = n^2$ ?