PROBLEM OF THE WEEK
Solution of Problem No. 1 (Spring 2005 Series)

**Problem:** Suppose \( b \) and \( c \) are real numbers randomly chosen in the interval \([0,1]\). What is the probability that the distance in the complex plane between the two roots of the equation \( z^2 + bz + c = 0 \) is not greater than 1?

**Solution** (by Georges Ghosn, Quebec, edited by the Panel)

The distance between the 2 roots, which is equal to \( \sqrt{\Delta} = \sqrt{b^2 - 4c} \), is not greater than 1 if and only if \(-1 \leq b^2 - 4c \leq 1\). That means that the point \( M(b,c) \) lies on the intersection of the region in between the 2 parabolas \( y = \frac{x^2 - 1}{4} \) and \( y = \frac{x^2 + 1}{4} \) and the square delimited by \( x = 0 \), \( x = 1 \), \( y = 0 \), \( y = 1 \).

The probability is equal to the area of this region, which is \( \int_0^1 \frac{x^2 + 1}{4} dx = \frac{1}{3} \), over the area of the square which is equal to 1. Consequently the probability is equal to \( \frac{1}{3} \).

Also solved by:

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