**Problem:** Let \( x_1 = \sqrt{2}, \ x_{n+1} = \sqrt{\frac{2x_n}{x_{n+1}}} \).
Prove that
\[
\prod_{n=1}^{\infty} x_n = \frac{\pi}{2}.
\]

**Solution** (by Georges Ghosn, Quebec; edited by the Panel)

All terms of the sequence \( x_n \) are positive, and therefore we can define the sequence \( y_n = \frac{1}{x_n} \) by:
\[
y_1 = \frac{\sqrt{2}}{2} \quad \text{and} \quad y_{n+1} = \sqrt{\frac{y_{n+1}}{2}}.
\]
Using the identity \( \cos 2x = 2\cos^2 x - 1 \), we get
\[
y_1 = \cos \frac{\pi}{2^2}, \quad y_2 = \sqrt{\frac{\cos \frac{\pi}{2^2} + 1}{2}} = \sqrt{\frac{2\cos^2 \frac{\pi}{2^3}}{2}} = \cos \frac{\pi}{2^3}.
\]
By induction, \( y_n = \sqrt{\cos \frac{\pi}{2^n} + 1} = \sqrt{\cos^2 \frac{\pi}{2^{n+1}}} = \cos \frac{\pi}{2^{n+1}} \).
Since \( \sin x \cos x = \frac{1}{2} \sin 2x \), we get
\[
y_1y_2 \ldots y_n \cdot \sin \frac{\pi}{2^{n+1}} = y_1 \ldots y_{n-1} \cdot \frac{1}{2} \sin \frac{\pi}{2^n} = \ldots = \frac{1}{2^n} \sin \frac{\pi}{2} = \frac{1}{2^n}.
\]
Hence
\[
x_1 \ldots x_n = \frac{1}{y_1 \ldots y_n} = 2^n \sin \frac{\pi}{2^{n+1}} = \frac{\pi}{2} \cdot \sin \left( \frac{\pi}{2^{n+1}} \right).
\]
Finally
\[
\prod_{n=1}^{\infty} x_n = \lim_{n \to \infty} \left( x_1 \ldots x_n \right) = \frac{\pi}{2} \cdot \lim_{n \to \infty} \frac{\sin \left( \frac{\pi}{2^{n+1}} \right)}{\frac{\pi}{2^{n+1}}} = \frac{\pi}{2}
\]
because \( \lim_{n \to \infty} \frac{\sin \frac{\pi}{2^{n+1}}}{\frac{\pi}{2^{n+1}}} = \lim_{x \to 0} \frac{\sin x}{x} = 1 \).

Also solved by:
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