**PROBLEM OF THE WEEK**
Solution of Problem No. 10 (Spring 2009 Series)

**Problem:** Let $Q$ be a convex quadrilateral each of whose sides has length at most 20. Show that if $O$ is an arbitrary interior point of $Q$, then at least one of the vertices of $Q$ has distance less than 15 from $O$.

**Solution** (by Xingyi Qin, Sr., Actuarial Science, Purdue University)

Suppose all vertices of $Q$ have distance of at least 15 from $O$. Use the Law of cosines:

$$
\cos \angle AOB = \frac{AO^2 + BO^2 - AB^2}{2 \cdot AO \cdot BO} \geq \frac{15^2 + 15^2 - 20^2}{2 \cdot 15 \cdot 15} = \frac{1}{9}
$$

$\Rightarrow \angle AOB \leq \arccos \frac{1}{9} < \frac{\pi}{2}$

For the same reason,

$$
\angle BOC < \frac{\pi}{2}, \quad \angle COD < \frac{\pi}{2}, \quad \angle DOA < \frac{\pi}{2}
$$

$\Rightarrow \angle AOB + \angle BOC + \angle COD + \angle DOA < \frac{\pi}{2} \cdot 4 = 2\pi$.

This is a contradiction. So the hypothesis is not valid, which means at least one of the vertices of $Q$ has distance less than 15 from $O$.

The problem was also solved by:

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