

PROBLEM OF THE WEEK
Solution of Problem No. 5 (Spring 2011 Series)

Problem: For any real numbers a, b with $a < b$, let $[a, b]$ denote the closed interval with end points a, b .

Given any finite collection of closed intervals

$$[a_1, b_1], \dots, [a_n, b_n]$$

such that any two of them have at least one point in common, show that there must be some point common to all the intervals.

Solution: (by Jorge Ramos, Sophomore, Physics, Purdue University)

Consider having n sets. I can label them set 1, set 2, \dots , set n such that $b_1 \leq b_2, \dots \leq b_n$. For any $2 \leq i \leq n$, $a_i \leq b_1$ because on the contrary there would not be a common point between set 1 and set i . Similarly, there would be an a_k ($1 \leq k \leq n$) such that for any $1 \leq w \leq n$, $a_w \leq a_k$. The interval $[a_k, b_1]$ will be common to all sets since all a 's are less than or equal to a_k and all b 's are greater than or equal to b_1 .

The problem was also solved by:

Undergraduates: Kaibo Gong (Math), Yixin Wang (So. ECE), Lifan Wu (So.)

Graduates: Pinaki Bhattacharya (Mech.E), Richard Eden (Math), Benjamin Philabaum (Phys.), Tairan Yuwen (Chemistry)

Others: Manuel Barbero (New York), Nicolas Busca (France), Hongwei Chen (Christopher Newport U. VA), Gruian Cornel (IT, Romania), Sandipan Dey (Graduate student, UMBC), Jonathan Dorfman (Bloomberg, LP, NY), Tom Engelsman (Chicago, IL), Elie Ghosn (Montreal, Quebec), Chris Kennedy (Christopher Newport Univ.), Steven Landy (IUPUI Physics staff), Kevin Laster (Indianapolis, IN), Paul Liu & Ron Estrin (Canada), Denes Molnar (Physics, Assistant Professor), Louis Rogliano (Corsica), Sorin Rubinstein (TAU faculty, Israel), Craig Schroeder (Ph.D. student, Stanford Univ.) Pawan Singh (Indianapolis), Steve Spindler (Chicago), Stephen Taylor (Bloomberg L.P. NY), Amitabha Tripathi (IIT, Delhi, India), William Wu (JPL), Shiju Zhang (Statistics faculty, St. Cloud State Univ.)