PROBLEM OF THE WEEK
Solution of Problem No. 13 (Spring 2012 Series)

Problem: A tetrahedron has a base which is an equilateral triangle of edge length one, and is placed so that the base is sitting flat on a table. The other three faces are congruent isosceles triangles with one edge an edge of the base (of course) and the other edges of length three. Find the length of the shortest path, lying in the union of the three isosceles faces, which starts and ends at the same vertex of the base, and which meets every line segment drawn from the top vertex to the perimeter of the base triangle.

Solution: (by Steven Landy, Physics Faculty, IUPUI)

Let the pyramid be labeled A, B, C, D (vertex). If we cut the base of the pyramid, and cut the remainder along AD, we can lay the lateral surface out flat producing a pentagon A, B, C, A'(which was the same as A), D. We wish to find the shortest distance from A to A'. This is the straight line joining these points whose length is given by $L = 2(3\sin 3\alpha)$ where $3\sin\alpha = 1/2$. (α is half the vertex angle of each isosceles triangle of the tetrahedron). Using the trig identity $\sin 3\alpha = 3\sin\alpha - 4\sin^3\alpha$, we get $L = \dfrac{26}{9}$.

The problem was also solved by:

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