PROBLEM OF THE WEEK

Solution of Problem No. 8 (Spring 2013 Series)

Problem:

Start with an even number of points (at least four points) in the plane, no three on the same straight line, half colored blue and half colored yellow. Show there is a straight line, which does not meet any of the points, which divides the points into two non-empty sets of points, both sets being half blue and half yellow.

Solution: (by Steven Landy, Physics Faculty, IUPUI)

Select any point P within the convex hull of the points, which is not one of the colored points and which is not on any line joining two of the colored points. Draw a line L through P which does not pass through any colored point. Attach a unit vector normal to L with its tail on L so as to indicate one of the two halves of the plane separated by L. Let $D = N(b) - N(y)$ be the number of blue points minus the number of yellow points in the indicated half plane. If $D = 0$, we are done. If not, rotate L counter clockwise around P by 180 degrees, causing D to change sign. Since during the rotation D changes in unit steps, there must have been some orientation in which $D = 0$. Because P is within the convex hull, each half plane is non-empty for any orientation. Thus the theorem is proved.

The problem was also solved by:

Graduates: Tairan Yuwen (Chemistry)

Others: Marco Biagini (Italy), Gruian Cornel (Cluj-Napoca, Romania), Hubert Desprez (Paris, France), Tom Engelsman (Tampa, FL), Massimo Frittelli (Italy), Lincoln James (HSE&Co. Chicago), Oliver Kroll (Stanford Law School), Jean Pierre Mutanguha (Student, Oklahoma Christian Univ.), Christopher Nelson (Graduate Student, UCSD), Craig Schroeder (Postdoc. UCLA), Steve Spindler (Chicago), Hao–Nhien Vu (Adjunct, Santa Ana College)