Problem:
How many ways are there to list 90 numbers consisting of ten ones, ten twos, ..., ten nines in a row so that for each \( j \), \( 1 \leq j \leq 9 \), no number bigger than \( j \) lies to the left of that \( j \) which is farthest to the left? Your answer should be in a fairly simple form.

Solution 1: (by David Stoner, High School Student, Aiken, South Carolina)

Consider the leftmost occurrence of each digit. These need to be in the order 1,2,3,...,9, which occurs with probability \( \frac{1}{9!} \) in a given random list. There are
\[
\binom{90}{10,10,10,10,10,10,10,10,10} = \frac{90!}{(10!)^9} \text{ lists, so } \frac{90!}{(10!)^99!} \text{ of them are valid.}
\]

Solution 2: (by Sorin Rubinstein, TAU Faculty, Tel Aviv, Israel)

We consider a horizontal list of 90 void entries which must be filled in. Firstly we fill in the ones. A 1 must be placed in the leftmost place in the list. The other ones may be filled in the list in
\[
\binom{89}{9} = \frac{89!}{9! \cdot 80!}
\]
ways. Then we fill in the twos. A 2 must be filled placed in the leftmost available (i.e. unoccupied) place. The other twos may be placed in the unoccupied 79 places in
\[
\binom{79}{9} = \frac{79!}{9! \cdot 70!}
\]
ways. Subsequently we fill in the threes, fours, and so on. There are:
\[
\frac{89!}{9! \cdot 80!} \cdot \frac{79!}{9! \cdot 70!} \cdot \frac{69!}{9! \cdot 60!} \cdots \frac{19!}{9! \cdot 10!} \cdot \frac{9!}{9! \cdot 0!}
\]
ways to fill in the whole list. Thus simplifies to:
\[
\frac{89!}{(9!)^9 \cdot 10^8 \cdot 8!}
\]

The problem was also solved by:

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