Problem No. 10 (Spring 2015 Series)

Let \( a \) be the number with a ternary expansion .1001100011. Show that \( a \) is the sum of two positive numbers which both have ternary expansions which use only zeros and twos. (These ternary expansions may have an infinite number of digits.)

[Remark: One way to work this problem, not the easiest way, is to show that any number in \([0,1]\) is the sum of two such numbers.]

A panel in the Mathematics Department publishes a challenging problem once a week and invites college & pre-college students, faculty, and staff to submit solutions. The objective of this is to stimulate and cultivate interest in good mathematics, especially among younger students. Solutions are due within two weeks from the date of publication.

Solutions can be emailed only as a pdf attachment to: sfchang@purdue.edu. Solutions can also be faxed to 765-496-3177 or sent by campus or U.S. mail to:

PROBLEM OF THE WEEK, 6th Floor, Math Sciences Bldg., Purdue Univ.,
150 North University St., West Lafayette, IN 47907-2067

Please include your name, address and status at your university or school on your problem solutions.

The names of those who submitted correct solutions will be posted on the Problem of the Week website and in the Math. Library, along with the best solution. Every Purdue student who submits three or more correct solutions will receive a Certificate of Merit. A prize fund of $300.00 will be distributed among those Purdue undergraduates who have contributed at least six correct solutions for the thirteen problems in the Spring 2015 series.