Problem No. 13 (Spring 2015 Series)

Let $Q^+$ denote the positive rationals. Let $f$ denote a function on $Q^+$ to $Q^+$ which satisfies the equation $f(u f(v)) = f(u)/v$ for all $u$ and $v$ in $Q^+$. Show that $f$ must be $1-1$ and onto, and that $f(uv) = f(u)f(v)$. Also give an explicit example of such a function. Hint: first show that $f(1) = 1$.