Problem:
Let $f(x)$ be a strictly increasing continuous function on a bounded interval $[a, b]$. Choose $c$ in $[a, b]$. Consider the two curvilinear triangles bounded by the vertical lines $x = a, x = b$, the horizontal line $y = f(c)$ and the graph of $f$. For which position $c$ is the sum of the areas of these curvilinear triangles minimal?

Solution by Hubert Desprez, Paris, France

First by $x \to \frac{x-a}{b-a}$ assume wlog $(a, b) = (0, 1)$. We have to minimize

$$
\varphi(c) = \int_0^c (f(c) - f) + \int_c^1 (f - f(c)) = (2c - 1)f(c) + \int_c^1 f + \int_0^c f, \text{ by monotony,}
$$

$$
\varphi(c) - \varphi(1/2) = 2 \left( (c - 1/2)f(c) - \int_{1/2}^c f \right) \geq 0, \text{ answer is } c = (a + b)/2.
$$

Remark from the panel: Solutions needed to apply to all strictly increasing functions, not just differentiable ones. For those who did assume differentiability we note that a strictly increasing differentiable function need not have a positive derivative at all points, as $x^3$ shows.

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