PROBLEM OF THE WEEK
Solution of Problem No. 6 (Spring 2015 Series)

Problem:
A ball is thrown straight up and caught at the same height at which it was released. Air resistance is a function of velocity. Did the ball spend more time during its flight going up or going down?

Solution by Adam Kline, Freshman, Chemistry & Physics, Purdue University

The force of air resistance is always opposite the direction of motion. Therefore, it does negative work on the ball as long as it is moving. The total (kinetic and potential) energy of the ball at a given point along its upward path is greater than the total energy of the ball at that same point along its downward path. Potential energy is a constant for any given point along the path, and so we have:

\[ K_{up}(h) + mgh > K_{down}(h) + mgh \]
\[ K_{up}(h) > K_{down}(h). \]

As kinetic energy is proportional to the square of velocity, the speed of the ball at any given point along the upward path must be greater than its speed at that point along the downward path.

\[ s_u(h) > s_d(h). \]

Since both paths are the same length and the speed is less at every point along the downwards path, the ball takes longer to come down than it does to go up.

The problem was also solved by:

Undergraduates: Victor Lee (Fr. CS), Bennett Marsh (Sr. Physics & Math)

Graduates: Tairan Yuwen (Chemistry)

Others: Hongwei Chen (Professor, Christopher Newport Univ. Virginia), Chung–Chin Jian (Postdoc, National Taiwan U), Steven Landy (Physics Faculty, IUPUI), Wei–Xiang Lien & Pan (Miaoli, Taiwan), Matthew Lim, Sorin Rubinstein (TAU faculty, Tel Aviv, Israel), Craig Schroeder (Postdoc. UCLA), David Stoner (HS Student, Aiken, S. Carolina)