PROBLEM OF THE WEEK
Solution of Problem No. 11 (Fall 2000 Series)

Problem: A particle moves in a vertical plane from rest under the influence of gravity and a force perpendicular to and proportional to its velocity. Obtain the equation of the trajectory, and identify the curve.

Solution (by Steven Landy, Fac. Physics at IUPUI)

Assume the mass is dropped from $x = y = 0$ at $t = 0$. Let $y =$ positive down. We then have $(F = kv)$

$$m\ddot{y} = mg - k\dot{x}, \quad m\ddot{x} = k\dot{y},$$

or

$$\ddot{y} = g - w\dot{x}, \quad \ddot{x} = w\dot{y}, \quad \text{where} \quad w = k/m.$$

Substituting gives $\dddot{y} = -w^2\dot{y}$,
or

$$\dot{y} = B\sin wt \Rightarrow y = -\frac{B}{w}\cos wt + \frac{B}{w},$$
$$\dot{x} = wB\sin wt \Rightarrow x = -\frac{B}{w}\sin wt + Bt.$$

Substitution in original differential equation gives $B = g/w$.

Finally

$$x = \frac{-g}{w^2}\sin wt + \frac{g}{w}t$$
$$y = \frac{-g}{w^2}\cos wt + \frac{g}{w^2}.$$

These determine a cycloid produced by a wheel of radius $R = g/w^2$ rolling on the x axis at speed $\frac{g}{w}$ where $w = k/m$.

Also solved by:

Undergraduates: Benjamin Zwickl (Fr. Phys)