PROBLEM OF THE WEEK Solution of Problem No. 13 (Fall 2000 Series)

Problem: Show that in the (x, y) plane, for odd integers A, B, C, the line Ax+By+C = 0 cannot intersect the parabola $y = x^2$ in a rational point.

Solution (by Steven Landy, Fac. Physics at IUPUI)

The x-coordinate of the point of intersection of the line and parabola is found from the equation

$$Ax + Bx^2 + C = 0.$$

Assume x is rational, x = p/q, where p, q are integers, not both even. Then

$$Apq + Bp^2 + Cq = 0.$$

If both p, q are odd, we have a contradiction, because the sum of three odd numbers cannot be zero. If p is even, q odd, we have again a contradiction. The same is true if p is odd and q is even. Hence, x cannot be rational.

Also solved by:

<u>Undergraduates</u>: Jeffrey D. Moser (Fr. MA/CS), Stevie Schraudner (Jr. CS)

Graduates: Vikram Buddhi (MA), Gajath Gunatillake (MA), Chris Lomont (MA)

Faculty: Sebastien Mercier (Research, Chem.), Ralph Shines (GAANN Fellow, MA)

<u>Others</u>: Damir D. Dzhafarov (Sr. Harrison H.S., WL), Jonathan Landy (Jr. Warren Central H.S., Indianapolis)

There was one incorrect solution.