## PROBLEM OF THE WEEK

Solution of Problem No. 13 (Fall 2000 Series)

Problem: Show that in the $(x, y)$ plane, for odd integers $A, B, C$, the line $A x+B y+C=0$ cannot intersect the parabola $y=x^{2}$ in a rational point.

Solution (by Steven Landy, Fac. Physics at IUPUI)
The $x$-coordinate of the point of intersection of the line and parabola is found from the equation

$$
A x+B x^{2}+C=0
$$

Assume $x$ is rational, $x=p / q$, where $p, q$ are integers, not both even. Then

$$
A p q+B p^{2}+C q=0
$$

If both $p, q$ are odd, we have a contradiction, because the sum of three odd numbers cannot be zero. If $p$ is even, $q$ odd, we have again a contradiction. The same is true if $p$ is odd and $q$ is even. Hence, $x$ cannot be rational.

Also solved by:
Undergraduates: Jeffrey D. Moser (Fr. MA/CS), Stevie Schraudner (Jr. CS)
Graduates: Vikram Buddhi (MA), Gajath Gunatillake (MA), Chris Lomont (MA)
Faculty: Sebastien Mercier (Research, Chem.), Ralph Shines (GAANN Fellow, MA)
Others: Damir D. Dzhafarov (Sr. Harrison H.S., WL), Jonathan Landy (Jr. Warren Central H.S., Indianapolis)

There was one incorrect solution.

