PROBLEM OF THE WEEK Solution of Problem No. 3 (Fall 2000 Series)

Problem: If a given equilateral triangle $\triangle$ of side $a$ can be covered by five equilateral triangles of side $b$, show that $\triangle$ can be covered by four of side $b$.

Solution (by Mike Hamburg, Jr. St. Joseph H.S., South Bend)
Suppose $\triangle$ can be covered by 5 equilateral triangles of side $b$ (henceforth " $b$-triangles"). Then we assert $a \leq 2 b$. For if $a>2 b$, then the vertices and midpoints of the sides of $\triangle$ ( 6 points at all) are mutually separated by $\frac{1}{2} a>b$. But no 2 points on a $b$-triangle are separated by a distance greater than $b$, hence no $b$-triangle can cover more than one of the 6 points.

But if $a \leq 2 b$ then $4 b$-triangles can be arranged to form a $2 b$-triangle which covers $\triangle$. Also solved by:

Undergraduates: Kevin Darkes (Soph. A\&AE), Haldun Kufluoglu (Sr. EE), James Lee (Sr. MA/CS), Stevie Schraudner (Jr. CS)

Graduates: Gajath Gunatillake (MA), Chris Lomont (MA)
Faculty \& Staff: Steven Landy (Phys. at IUPUI)
Others: Damir Dzhafarov, Jake Foster (Sr. \& Soph., resp., Harrison H.S., WL)
There was one incorrect solution.

