## PROBLEM OF THE WEEK Solution of Problem No. 3 (Fall 2000 Series)

**Problem:** If a given equilateral triangle  $\triangle$  of side *a* can be covered by five equilateral triangles of side *b*, show that  $\triangle$  can be covered by four of side *b*.

Solution (by Mike Hamburg, Jr. St. Joseph H.S., South Bend)

Suppose  $\triangle$  can be covered by 5 equilateral triangles of side *b* (henceforth "*b*-triangles"). Then we assert  $a \leq 2b$ . For if a > 2b, then the vertices and midpoints of the sides of  $\triangle$  (6 points at all) are mutually separated by  $\frac{1}{2}a > b$ . But no 2 points on a *b*-triangle are separated by a distance greater than *b*, hence no *b*-triangle can cover more than one of the 6 points.

But if  $a \leq 2b$  then 4 *b*-triangles can be arranged to form a 2*b*-triangle which covers  $\triangle$ .

Also solved by:

<u>Undergraduates</u>: Kevin Darkes (Soph. A&AE), Haldun Kufluoglu (Sr. EE), James Lee (Sr. MA/CS), Stevie Schraudner (Jr. CS)

Graduates: Gajath Gunatillake (MA), Chris Lomont (MA)

Faculty & Staff: Steven Landy (Phys. at IUPUI)

Others: Damir Dzhafarov, Jake Foster (Sr. & Soph., resp., Harrison H.S., WL)

There was one incorrect solution.