PROBLEM OF THE WEEK

Solution of Problem No. 4 (Fall 2000 Series)

Problem: Let $x_1, x_2, \cdots, x_n$ be n points in space. Between any pair (x_i, x_j) there is an arrow either from x_i to x_j or from x_j to x_i (this is a "complete oriented graph of size n"). Show that there is a path $x_{a_1} \to x_{a_2} \to \cdots \to x_{a_n}$ which includes all of $x_1, \ldots, x_n$ and proceeds in the direction of the arrows.

Solution (by the Panel)

Proof by induction on n. The assertion is trivial for $n = 1$ and $n = 2$. Assume it is true for all $k < n$. Choose any k, $1 < k < n$. Let A be the set of i for which $x_i \to x_k$, and B the set of i for which $x_k \to x_i$. By the induction assumption the $\{x_i\}$ with $i \in A$ can be arranged as $\{x_{a_i}\}$ so that $x_{a_1} \to x_{a_2} \to \cdots \to x_{a_{k-1}}$; likewise the set $\{x_i\}$ with $i \in B$ can be arranged so that $x_{a_{k+1}} \to \cdots \to x_{a_n}$. Then $x_{a_1} \to x_{a_2} \to \cdots \to x_{a_k} \to \cdots \to x_{a_n}$ is the desired path.

Solved by:

Undergraduates: Kevin Darkes (Soph. A&AE), James Lee (Sr. MA/CS), Yee-Ching Yeow (Jr. Math)

Graduates: Vikram Buddhi (MA), Yalin Firat Celikler (MA), Gajath Gunatillake (MA), Wook Kim (MA), Chris Lomont (MA), Mohammed Majidi (MA visitor)

Faculty: Steven Landy (Phys. at IUPUI)

Others: Damir D. Dzhafarov (Sr. Harrison H.S., Laf), Jake Foster (Soph. Harrison H.S., WL), Mike Hamburg (Jr. St. Joseph H.S., South Bend)

There was one incorrect solution.