

PROBLEM OF THE WEEK
Solution of Problem No. 5 (Fall 2000 Series)

Problem: Prove that the polynomials

$P_n(x) = x^{2n} - 2x^{2n-1} + 3x^{2n-2} - \dots - 2nx + 2n + 1$ ($n = 1, 2, \dots$) have no real zero.

Solution (by Vikram Buddhi, Gr. Math)

Clearly $P_n(x) > 0$ for $x \leq 0$, since the terms with negative coefficients are multiplied by odd powers of x . Now

$$P_n(x) + xP_n(x) = x(x^{2n} - x^{2n-1} + x^{2n-2} \dots - x + 1) + 2n + 1$$

so

$$P_n(x) > x \frac{x^{2n+1} + 1}{x + 1} + 2n + 1$$
$$P_n(x) > 2n + 1 \quad \text{for } x > 0.$$

Also solved by:

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Two unacceptable solutions were received.