PROBLEM OF THE WEEK

Solution of Problem No. 9 (Fall 2000 Series)

Problem: Define a strip S to be the open set of points in the plane lying between two parallel lines. Let $|S|$ be the width of S. Given an infinite sequence $\{S_i\}$ of strips, show that there are points in the plane that are not in any of the S_i if $\Sigma|S_i|$ converges.

Solution (by Steven Landy, Fac. Physics at IUPUI)

Let $\Sigma_1^\infty|S_i| = w$. Consider the intersection of the union of the strips with a circular disk of radius R. Each strip S_i intersects the disk with a length $\leq 2R$. So the area of the intersection is $\leq 2R|S_i|$ and the area of the intersection of $\cup S_i$ with the disk $\leq 2R\Sigma|S_i| \leq 2Rw$. Choose $R > 2w/\pi$ then the area of the circle in $R^2\pi > 2wR \geq 2R\Sigma|S_i|$, so some of the points of the disk are not in any of the S_i.

Also solved by:

Graduates: Gajath Gunatillake (MA)

Others: Mike Hamburg (Jr. St. Joseph's H.S., South Bend)

There was one unacceptable solution.